MIT Kavli Institute **Chandra X-Ray Center**

MEMORANDUM
December 9, 2016

To:	Jonathan McDowell, SDS Group Leader
From:	Glenn E. Allen, SDS
Subject:	`mkacisrmf` spec
Revision:	1.00
URL:	http://space.mit.edu/CXC/docs/docs.html#mkacisrmf
File:	/nfs/inconceivable/d0/sds/specs/mkacisrmf/mkacisrmf_spec_1.00.tex

1 Description

This spec, which is incomplete, describes how **`mkacisrmf`** computes an ACIS PHA RMF for a single region.

2 Parameters

1. `infile`,f,a,“ ”,,,“Input CALDB file”
2. `energy`,s,a,” “,,,”Energy grid in keV”
3. `channel`,s,a,” “,,,”Channel grid”

3 Error checking

1. `infile`:
   (a) Existence:
   If the `infile` does not exist, then **`mkacisrmf`** exits with an error message.
   (b) Permission:
   If the `infile` exists, but the file permissions do not allow it to be read, then **`mkacisrmf`** exits with an error message.
   (c) Ideal front-illuminated RMF:
      i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \text{CDB\_ACIS\_RESPONSE} \quad (1)$$

      that is appropriate for the front-illuminated CCDs, then **`mkacisrmf`** exits with an error message. Hereafter, this HDU is refered to as $h_{\rm ideal,fi}$.

ii. If the `infile` contains an HDU $h_{\mathrm{ideal,fi}}$, but the binary table does not include the columns `ENERGY`, `NGRP`, `PHABOUND`, `AVERESP`, and `SHIFT`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{ideal,fi}}$, but the header does not include the keywords `GAIN0`, `GAIN00`, `GAIN01`, `GAIN02`, `GAIN03`, `GAIN10`, `GAIN11`, `GAIN12`, `GAIN13`, `GAIN20`, `GAIN21`, `GAIN22`, `GAIN23`, `GAIN30`, `GAIN31`, `GAIN32`, `GAIN33`, `GAIN40`, `GAIN41`, `GAIN42`, `GAIN43`, `GAIN60`, `GAIN61`, `GAIN62`, `GAIN63`, `GAIN80`, `GAIN81`, `GAIN82`, `GAIN83`, `GAIN90`, `GAIN91`, `GAIN92`, `GAIN93`, `SHIFT00`, `SHIFT01`, `SHIFT02`, `SHIFT03`, `SHIFT10`, `SHIFT11`, `SHIFT12`, `SHIFT13`, `SHIFT20`, `SHIFT21`, `SHIFT22`, `SHIFT23`, `SHIFT30`, `SHIFT31`, `SHIFT32`, `SHIFT33`, `SHIFT40`, `SHIFT41`, `SHIFT42`, `SHIFT43`, `SHIFT60`, `SHIFT61`, `SHIFT62`, `SHIFT63`, `SHIFT80`, `SHIFT81`, `SHIFT82`, `SHIFT83`, `SHIFT90`, `SHIFT91`, `SHIFT92`, and `SHIFT93`, then `mkacisrmf` exits with an error message.

(d) Ideal back-illuminated RMF:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESPONSE} \quad (2)$$

that is appropriate for the back-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{ideal,bi}}$.[1]

ii. If the `infile` contains an HDU $h_{\mathrm{ideal,bi}}$, but the binary table does not include the columns `ENERGY`, `NGRP`, `PHABOUND`, `AVERESP`, and `SHIFT`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{ideal,bi}}$, but the header does not include the keywords `GAIN0`, `GAIN50`, `GAIN51`, `GAIN52`, `GAIN53`, `GAIN70`, `GAIN71`, `GAIN72`, `GAIN73`, `SHIFT50`, `SHIFT51`, `SHIFT52`, `SHIFT53`, `SHIFT70`, `SHIFT71`, `SHIFT72`, and `SHIFT73`, then `mkacisrmf` exits with an error message.

(e) Front-illuminated scatter matrix data:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESP\_CTI} \quad (3)$$

that is appropriate for the front-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{scatter,fi}}$.

ii. If the `infile` contains an HDU $h_{\mathrm{scatter,fi}}$, but the binary table does not include the columns `CCD_ID`, `CHIPX_LO`, `CHIPX_HI`, `CHIPY_LO`, `CHIPY_HI`, `PHACHAN`, `L1_WIDTH`, and `L1_POS`, then `mkacisrmf` exits with an error message.

iii. If the `infile` contains an HDU $h_{\mathrm{scatter,fi}}$, but the header does not include the keywords `L1ALPH1` and `L1ALPH2`, then `mkacisrmf` exits with an error message.

(f) Back-illuminated scatter matrix data:

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \mathrm{CDB\_ACIS\_RESP\_CTI} \quad (4)$$

that is appropriate for the back-illuminated CCDs, then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as $h_{\mathrm{scatter,bi}}$.[2]

ii. If the `infile` contains an HDU $h_{\mathrm{scatter,bi}}$, but the binary table does not include the columns `CCD_ID`, `CHIPX_LO`, `CHIPX_HI`, `CHIPY_LO`, `CHIPY_HI`, `PHACHAN`, `G1_FWHM`, `G1_POS`, `G1_AMPL`, `G2_FWHM`, `G2_POS`, and `G2_AMPL`, then `mkacisrmf` exits with an error message.

(g) Gain tweak data:

[1] Hereafter h_{ideal} is used to generically refer to $h_{\mathrm{ideal,fi}}$ for a front-illuminated CCD or $h_{\mathrm{ideal,bi}}$ for a back-illuminated CCD.

[2] Hereafter h_{scatter} is used to generically refer to $h_{\mathrm{scatter,fi}}$ for a front-illuminated CCD or $h_{\mathrm{scatter,bi}}$ for a back-illuminated CCD.

i. If the `infile` does not contain an HDU with

$$\texttt{CONTENT} = \text{CDB\_ACIS\_RESP\_GCORR}, \tag{5}$$

then `mkacisrmf` exits with an error message. Hereafter, this HDU is refered to as h_{gtweak}.

ii. If the `infile` contains an HDU h_{gtweak}, but the binary table does not include the columns `ENERGY`, `GTWEAK0`, `GTWEAK1`, `GTWEAK2`, `GTWEAK3`, `GTWEAK4`, `GTWEAK5`, `GTWEAK6`, `GTWEAK7`, `GTWEAK8`, and `GTWEAK9`, then `mkacisrmf` exits with an error message.

2. `energy`:

(a) Number:
If N_{energy}, the number of elements in the input `energy` grid, is less than two, then `mkacisrmf` exits with an error message.

(b) Minimum:
If `energy`[0], the first element in the `energy` grid, is less than `ENERGY`[0], where `ENERGY`[0] is the value in the first row of the column `ENERGY` of HDU h_{ideal}, then `mkacisrmf` exits with an error message.

(c) Maximum:
If `energy`$[N_{\text{energy}} - 1]$, the last element in the `energy` grid, is greater than `ENERGY`$[N_{\text{ideal}} - 1]$, where `ENERGY`$[N_{\text{ideal}} - 1]$ is the value in the last row of the column `ENERGY` of HDU h_{ideal}, then `mkacisrmf` exits with an error message.

3. `channel`:

(a) Number:
If N_{channel}, the number of elements in the input `channel` grid, is less than two, then `mkacisrmf` exits with an error message.

(b) Minimum:
If `channel`[0], the first element in the `channel` grid, is less than 1, then `mkacisrmf` exits with an error message.

(c) Maximum:
If `channel`$[N_{\text{channel}}-1]$, the last element in the `channel` grid, is greater than 4096, then `mkacisrmf` exits with an error message.

4 Processing

An RMF is a two-dimensional array $R[i, j]$ with $N_{\text{energy}} - 1$ energy elements i and N_{channel} elements j. Here, $N_{\text{channel}} = 4096$. The three basic steps to computing an RMF, which are described hereafter, are: (1) obtaining a "position-independent,[3]" ideal RMF, (2) obtaining a position-dependent scatter matrix, and (3) convolving the ideal RMF and scatter matrix.

The steps in sections 4.1–4.3 are performed for each `energy` index $i = 0, 1, \ldots, N_{\text{energy}} - 2$.

4.1 Ideal RMF

To obtain the "ideal" or "pre-CTI," response for `energy` bin i:

1. The mean `energy` is computed:

$$E_{\text{mean}}[i] = \frac{\texttt{energy}[i] + \texttt{energy}[i+1]}{2}. \tag{6}$$

[3] There is one ideal front-illuminated RMF and one ideal back-illuminated RMF.

2. The mean `energy` is tweaked:

   (a) If

$$E_{\text{mean}}[i] \quad < \quad \texttt{ENERGY}[0], \tag{7}$$

   where $\texttt{ENERGY}[0]$ is the value in the first row of the column `ENERGY` in HDU h_{gtweak}, then

$$E_{\text{tweaked}}[i] \quad = \quad E_{\text{mean}}[i]\,\texttt{GTWEAK}c[0], \tag{8}$$

   where $c = 0,\ 1,\ \ldots,$ or 9, depending on the value of the `CCD_ID` of the region, and $\texttt{GTWEAK}c$ is a column in HDU h_{gtweak}.

   (b) If

$$E_{\text{mean}}[i] \quad \geq \quad \texttt{ENERGY}[N_{\text{gtweak}} - 1], \tag{9}$$

   where N_{gtweak} is the number of rows in the column $\texttt{GTWEAK}c$, then

$$E_{\text{tweaked}}[i] \quad = \quad E_{\text{mean}}[i]\,\texttt{GTWEAK}c[N_{\text{gtweak}} - 1]. \tag{10}$$

   (c) If there is a row r such that

$$E_{\text{mean}}[i] \quad \geq \quad \texttt{ENERGY}[r] \text{ and} \tag{11}$$
$$E_{\text{mean}}[i] \quad < \quad \texttt{ENERGY}[r+1], \tag{12}$$

   then

$$E_{\text{tweaked}}[i] \quad = \quad E_{\text{mean}}[i]\left(\left[\frac{E_{\text{mean}}[i] - \texttt{ENERGY}[r]}{\texttt{ENERGY}[r+1] - \texttt{ENERGY}[r]}\right]\left(\texttt{GTWEAK}c[r+1] - \texttt{GTWEAK}c[r]\right) + \tag{13}$$
$$\texttt{GTWEAK}c[r]\right). \tag{14}$$

3. The ideal response is computed:

   (a) The row of HDU h_{ideal} corresponding to $E_{\text{tweaked}}[i]$ is found:

      i. If

$$E_{\text{tweaked}}[i] \quad < \quad \texttt{ENERGY}[0], \tag{15}$$

      where $\texttt{ENERGY}[0]$ is the value in the first row of the column `ENERGY` of HDU h_{ideal}, then the row

$$r \quad = \quad 0 \tag{16}$$

      and the weight for this row

$$w \quad = \quad 1. \tag{17}$$

      ii. If

$$E_{\text{tweaked}}[i] \quad \geq \quad \texttt{ENERGY}[N_{\text{ideal}} - 1], \tag{18}$$

      then

$$r \quad = \quad N_{\text{ideal}} - 2 \text{ and} \tag{19}$$
$$w \quad = \quad 0. \tag{20}$$

iii. If

$$E_{\rm tweaked}[i] \geq \texttt{ENERGY}[0] \text{ and} \tag{21}$$
$$E_{\rm tweaked}[i] < \texttt{ENERGY}[N_{\rm ideal} - 1], \tag{22}$$

then r is the row such that

$$E_{\rm tweaked}[i] \geq \texttt{ENERGY}[r] \text{ and} \tag{23}$$
$$E_{\rm tweaked}[i] < \texttt{ENERGY}[r+1] \tag{24}$$

and the weight

$$w = \frac{\texttt{ENERGY}[r+1] - E_{\rm tweaked}[i]}{\texttt{ENERGY}[r+1] - \texttt{ENERGY}[r]}. \tag{25}$$

Items 4.1.3(a)i and 4.1.3(a)ii limit the energy range over which it is possible to create an RMF.[4]

(b) The mean PHA for each group in this row is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$, where $\texttt{NGRP}[r]$ is the value in row r of the column NGRP of HDU $h_{\rm ideal}$:

i. Lower limit on PHA for group m:

$$j_{\rm lo}[m] = \text{int}\left(\texttt{PHABOUND}[r,m]\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn\right), \tag{26}$$

where $\texttt{PHABOUND}[r,m]$ is the $m^{\rm th}$ element of row r of the column PHABOUND of HDU $h_{\rm ideal}$ and GAIN0, GAINcn, and SHIFTcn are keywords in the same HDU. Again, $c = 0, 1, \ldots,$ or 9 and $n = 0, 1, 2,$ or 3, depending on the values of the CCD_ID and NODE_ID[5] of the region, respectively. If

$$j_{\rm lo}[m] < 1, \tag{27}$$

then

$$j_{\rm lo}[m] = 1. \tag{28}$$

ii. Upper limit on PHA for group m:

$$j_{\rm hi}[m] = \text{int}\left(\left(\texttt{PHABOUND}[r,m+1] - 1\right)\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn\right). \tag{29}$$

If

$$j_{\rm hi}[m] > 4096, \tag{30}$$

then

$$j_{\rm hi}[m] = 4096. \tag{31}$$

iii. Mean PHA for group m:

$$j_{\rm mean}[m] = \left(\frac{\texttt{PHABOUND}[r,m] + \texttt{PHABOUND}[r,m+1] - 1}{2}\right)\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right) + \texttt{SHIFT}cn. \tag{32}$$

[4] For the CALDB file acisD2000-01-29p2_respN0006.fits, this range is 0.243–12.0 keV.

[5] The NODE_ID = 0, 1, 2, and 3 for CHIPX = 1–256, 257–512, 513–768, and 769–1024, respectively.

(c) The ideal response for this row is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$:

i. If

$$\begin{aligned} m &= 0 \text{ or} & (33)\\ m &= \texttt{NGRP}[r] - 2 \text{ or} & (34)\\ j_{\rm lo}[m] &= j_{\rm hi}[m], & (35)\end{aligned}$$

then, for $j = j_{\rm lo}[m], j_{\rm lo}[m]+1, \ldots, j_{\rm hi}[m]$,

$$\mathcal{R}_r[j-1] = \texttt{AVERESP}[r, m], \tag{36}$$

where $\texttt{AVERESP}[r, m]$ is the $m^{\rm th}$ element of the column AVERESP in row r of HDU $h_{\rm ideal}$.

ii. If

$$\begin{aligned} m &> 0 \text{ and} & (37)\\ m &< \texttt{NGRP}[r] - 2 \text{ and} & (38)\\ j_{\rm lo}[m] &< j_{\rm hi}[m], & (39)\end{aligned}$$

then, for $j = j_{\rm lo}[m], j_{\rm lo}[m]+1, \ldots, \mathrm{int}\big((j_{\rm lo}[m] + j_{\rm hi}[m])\,/2\big) - 1$,

$$\begin{aligned} \mathcal{R}_r[j-1] &= \left(\frac{j - j_{\rm mean}[m-1]}{j_{\rm mean}[m] - j_{\rm mean}[m-1]}\right)\Big(\texttt{AVERESP}[r, m] - \texttt{AVERESP}[r, m-1]\Big) + & (40)\\ &\quad \texttt{AVERESP}[r, m-1] & (41)\end{aligned}$$

and for $j = \mathrm{int}\left((j_{\rm lo}[m] + j_{\rm hi}[m])\,/2\right), \mathrm{int}\left((j_{\rm lo}[m] + j_{\rm hi}[m])\,/2\right) + 1, \ldots, j_{\rm hi}[m]$,

$$\begin{aligned} \mathcal{R}_r[j-1] &= \left(\frac{j - j_{\rm mean}[m]}{j_{\rm mean}[m+1] - j_{\rm mean}[m]}\right)\Big(\texttt{AVERESP}[r, m+1] - \texttt{AVERESP}[r, m]\Big) + & (42)\\ &\quad \texttt{AVERESP}[r, m]. & (43)\end{aligned}$$

(d) The ideal response for this row is normalized:

i.

$$\mathcal{R}_{r,\rm tot} = 0. \tag{44}$$

ii. For $j = j_{\rm lo}[0], j_{\rm lo}[0]+1, \ldots, j_{\rm hi}[\texttt{NGRP}[r]-2]$,

$$\mathcal{R}_{r,\rm tot} = \mathcal{R}_{r,\rm tot} + \mathcal{R}_r[j-1]. \tag{45}$$

iii. For $j = j_{\rm lo}[0], j_{\rm lo}[0]+1, \ldots, j_{\rm hi}[\texttt{NGRP}[r]-2]$,

$$\mathcal{R}_r[j-1] = \frac{\mathcal{R}_r[j-1]}{\mathcal{R}_{r,\rm tot}}. \tag{46}$$

(e) The ideal response for the next row is computed:
Steps 4.1.3b–4.1.3d are repeated to compute $\mathcal{R}_{r+1}$ for row $r+1$.

4. The pulse height shift is computed:
For $m = 0, 1, \ldots, \texttt{NGRP}[r] - 2$,

(a) For $j = \texttt{PHABOUND}[r, m], \texttt{PHABOUND}[r, m]+1, \ldots, \texttt{PHABOUND}[r, m+1]$,

$$\begin{aligned} s[j-1] &= 0.01\left(\frac{\texttt{GAIN0}}{\texttt{GAIN}cn}\right)\left[\left(\frac{\texttt{PHABOUND}[r, m+1] - j}{\texttt{PHABOUND}[r, m+1] - \texttt{PHABOUND}[r, m]}\right)\texttt{SHIFT}[r, m] + \right.\\ &\quad \left.\left(\frac{j - \texttt{PHABOUND}[r, m]}{\texttt{PHABOUND}[r, m+1] - \texttt{PHABOUND}[r, m]}\right)\texttt{SHIFT}[r, m+1]\right], & (47)\end{aligned}$$

where $\texttt{SHIFT}[r, m]$ is the $m^{\rm th}$ element of row r of the column SHIFT in HDU $h_{\rm ideal}$.

5. The response $\mathcal{R}$ is obtained from w, $\mathcal{R}_r$, $\mathcal{R}_{r+1}$, and s:
   For $j = 1, 2, \ldots, 4096$,

   (a) If

$$s[j-1] < 0.1, \tag{48}$$

   then the shifted pulse heights

$$j_- = j \text{ and} \tag{49}$$
$$j_+ = j, \tag{50}$$

   the fractional amount of the shifted pulse heights

$$\Delta_- = 0 \text{ and} \tag{51}$$
$$\Delta_+ = 0, \tag{52}$$

   and the out-of-bounds flags

$$f_- = 1 \text{ and} \tag{53}$$
$$f_+ = 1. \tag{54}$$

   (b) If

$$s[j-1] \geq 0.1, \tag{55}$$

   then

   i.

$$x_- = j - (1-w)\, s[j-1] \tag{56}$$
$$x_+ = j + ws[j-1] \tag{57}$$
$$j_- = \text{int}\,(x_-) \tag{58}$$
$$j_+ = \text{int}\,(x_+) \tag{59}$$
$$\Delta j_- = x_- - j_- \tag{60}$$
$$\Delta j_- = x_+ - j_+ \tag{61}$$

   ii. If

$$j_- \geq 1 \text{ and} \tag{62}$$
$$j_- \leq 4095, \tag{63}$$

   then

$$f_- = 1. \tag{64}$$

   iii. If

$$j_- < 1 \text{ or} \tag{65}$$
$$j_- > 4095, \tag{66}$$

   then

$$f_- = 0. \tag{67}$$

iv. If

$$j_+ \geq 1 \text{ and} \tag{68}$$
$$j_+ \leq 4095, \tag{69}$$

then

$$f_+ = 1. \tag{70}$$

v. If

$$j_+ < 1 \text{ or} \tag{71}$$
$$j_+ > 4095, \tag{72}$$

then

$$f_+ = 0. \tag{73}$$

(c) The shifted and interpolated response

$$\mathcal{R}[j-1] = \frac{1}{f_- w + f_+(1-w)}\Big[f_- w\Big((1-\Delta j_-)\,\mathcal{R}_r[j_- - 1] + \Delta j_- \mathcal{R}_r[j_-]\Big) + \tag{74}$$
$$f_+(1-w)\Big((1-\Delta j_+)\,\mathcal{R}_{r+1}[j_+ - 1] + \Delta j_+ \mathcal{R}_{r+1}[j_+]\Big)\Big]. \tag{75}$$

4.2 Scatter matrix

To obtain a position-dependent scatter matrix $S[m,k]$:[6]

1. The scatter matrix data are read:

   (a) Front-illuminated CCD:
   If the `CCD_ID` corresponds to a front-illuminated CCD, then for $m = 0,\ 1,\ \ldots,\ N_{\rm phachan} - 1$,[7]

   i. At and below the peak of the scatter matrix function:
   For $k = -500,\ -499,\ \ldots,\ \mathrm{int}(\texttt{L1\_POS}[m])$,

$$\mathcal{S}[m, k+500] = -\left|\texttt{L1ALPH1}\right| \log\left(1 + \left(\frac{k - \texttt{L1\_POS}[m]}{\texttt{L1\_WIDTH}[m]}\right)^2\right). \tag{76}$$

   ii. Above the peak of the scatter matrix function:
   For $k = \mathrm{int}(\texttt{L1\_POS}) + 1,\ \mathrm{int}(\texttt{L1\_POS}[m]) + 2,\ \ldots,\ 500$,

$$\mathcal{S}[m, k+500] = \left|\texttt{L1ALPH2}\right| \log\left(1 + \left(\frac{k - \texttt{L1\_POS}[m]}{\texttt{L1\_WIDTH}[m]}\right)^2\right). \tag{77}$$

   `L1ALPH1` and `L1ALPH2` are keywords and `L1_POS`, `L1_WIDTH`, and `PHACHAN` are columns in HDU $h_{\rm scatter,fi}$ of the `infile`. The logarithm is used so that the contour interpolation in section 4.2.2 is performed on the log of scattering matrix function. The sign change of $\mathcal{S}$ from one side of the peak (eqn. 76) to the other (eqn. 77) is also important for contour interpolation.

[6] There are $N_{\rm phachan}$ pulse height channels m and $N_{\rm scatter} = 1001$ scatter channels k. The number of scatter channels is hard coded.

[7] For HDU $h_{\rm scatter,fi}$ of the CALDB file acisD2000-01-29p2_respN0006.fits, the $N_{\rm phachan} = 16$ values of `PHACHAN`$[m]$ are 40, 60, 80, 100, 128, 180, 230, 300, 380, 460, 512, 670, 1025, 1535, 2050, and 3100.

(b) Back-illuminated CCD:
If the CCD_ID corresponds to a back-illuminated CCD, then for $m = 0, 1, \ldots, N_{\text{phachan}} - 1$,[8]

i. If

$$\texttt{G1\_FWHM}[m] \quad < \quad 1.5, \tag{78}$$

then

$$\texttt{G1\_AMPL}[m] \quad = \quad \texttt{G1\_AMPL}[m] \left(\frac{\texttt{G1\_FWHM}[m]}{1.5} \right) \quad \text{and} \tag{79}$$

$$\texttt{G1\_FWHM}[m] \quad = \quad 1.5. \tag{80}$$

ii. At and below the peak of the scatter matrix function:
For $k = -500, -499, \ldots, \text{int}(\texttt{G1\_POS}[m])$,

$$\mathcal{S}[m, k + 500] \quad = \quad -\frac{1}{2} \left((k - \texttt{G1\_POS}[m]) \frac{2.35482}{\texttt{G1\_FWHM}[m]} \right)^2. \tag{81}$$

iii. Above the peak of the scatter matrix function:
For $k = \text{int}(\texttt{G1\_POS}) + 1, \text{int}(\texttt{G1\_POS}[m]) + 2, \ldots, 500$,

$$\mathcal{S}[m, k + 500] \quad = \quad \frac{1}{2} \left((k - \texttt{G1\_POS}[m]) \frac{2.35482}{\texttt{G1\_FWHM}[m]} \right)^2. \tag{82}$$

G1_AMPL, G1_FWHM, G1_POS, and PHACHAN are columns in HDU $h_{\text{scatter,bi}}$ of the `infile`.[9] Again, equations 81 and 82 are logarithms of the scatter matrix function and the sign change above the peak is intentional.

2. The scatter matrix is (contour) interpolated to the `channel` grid:

(a) Front-illuminated CCD:
If the CCD_ID corresponds to a front-illuminated CCD, then for $j = 1, 2, \ldots, 4096$,

i. If

$$j \quad < \quad \texttt{PHACHAN}[0], \tag{83}$$

then

$$S[j - 1, k + 500] \quad = \quad \mathcal{S}[0, k + 500] \tag{84}$$

for $k = -500, -499, \ldots, 500$.

ii. If

$$j \quad \geq \quad \texttt{PHACHAN}[N_{\text{phachan}} - 1], \tag{85}$$

then

$$S[j - 1, k + 500] \quad = \quad \mathcal{S}[\texttt{PHACHAN}[N_{\text{phachan}} - 1] - 1, k + 500] \tag{86}$$

for $k = -500, -499, \ldots, 500$.

[8]For HDU $h_{\text{scatter,bi}}$ of the CALDB file acisD2000-01-29p2_respN0006.fits, the $N_{\text{phachan}} = 20$ values of PHACHAN$[m]$ are 40, 60, 80, 100, 114, 128, 142, 156, 170, 180, 230, 300, 380, 460, 512, 670, 1025, 1535, 2050, and 3100.

[9]The data in the columns G2_AMPL, G2_FWHM, and G2_POS of HDU $h_{\text{scatter,bi}}$ of the `infile` are not used because the values of G2_AMPL$[m] = 0$ for all m, at least for the CALDB file acisD2000-01-29p2_respN0006.fits.

iii. If there is some m such that

$$j \geq \texttt{PHACHAN}[m] \text{ and} \quad (87)$$
$$j < \texttt{PHACHAN}[m+1], \quad (88)$$

then

$$S[j-1, k+500] = \left(\frac{j - \texttt{PHACHAN}[m]}{\texttt{PHACHAN}[m+1] - \texttt{PHACHAN}[m]}\right) \times \quad (89)$$
$$\left(\mathcal{S}[m+1, k+500] - \mathcal{S}[m, k+500]\right) + \mathcal{S}[m, k+500] \quad (90)$$

for $k = -500, -499, \ldots, 500$.

iv. At and below the peak of the scatter matrix function:
For $k = -500, -499, \ldots, \text{int}(\texttt{L1\_POS}[m])$,

$$S[j-1, k+500] = e^{S[j-1,k+500]}. \quad (91)$$

The use of the exponential function removes the use of the logarithm in equation 76.

v. Above the peak of the scatter matrix function:
For $k = \text{int}(\texttt{L1\_POS}) + 1, \text{int}(\texttt{L1\_POS}[m]) + 2, \ldots, 500$,

$$S[j-1, k+500] = e^{-S[j-1,k+500]}. \quad (92)$$

The sign change in equation 105 relative to equation 104 removes the sign change in equation 77 relative to equation 76.

(b) Back-illuminated CCD:
If the CCD_ID corresponds to a back-illuminated CCD, then for $j = 1, 2, \ldots, 4096$,

i. If

$$j < \texttt{PHACHAN}[0], \quad (93)$$

then

$$S[j-1, k+500] = \log(\texttt{G1\_AMPL}[0]) + \mathcal{S}[0, k+500] \quad (94)$$

for $k = -500, -499, \ldots, 500$.

ii. If

$$j \geq \texttt{PHACHAN}[N_{\text{phachan}} - 1], \quad (95)$$

then

$$S[j-1, k+500] = \log(\texttt{G1\_AMPL}[N_{\text{phachan}} - 1]) + \quad (96)$$
$$\mathcal{S}[\texttt{PHACHAN}[N_{\text{phachan}} - 1] - 1, k+500] \quad (97)$$

for $k = -500, -499, \ldots, 500$.

iii. If there is some m such that

$$j \geq \texttt{PHACHAN}[m] \text{ and} \quad (98)$$
$$j < \texttt{PHACHAN}[m+1], \quad (99)$$

then

$$S[j-1,k+500] = \left(\frac{j-\texttt{PHACHAN}[m]}{\texttt{PHACHAN}[m+1]-\texttt{PHACHAN}[m]}\right)\times \tag{100}$$

$$\left[\left(\texttt{G1\_AMPL}[m+1,k+500]-\texttt{G1\_AMPL}[m,k+500]\right)+ \tag{101}$$

$$\left(\mathcal{S}[m+1,k+500]-\mathcal{S}[m,k+500]\right)\right]+ \tag{102}$$

$$\texttt{G1\_AMPL}[m,k+500]+\mathcal{S}[m,k+500] \tag{103}$$

for $k=-500,\,-499,\,\ldots,\,500$.

iv. At and below the peak of the scatter matrix function:
For $k=-500,\,-499,\,\ldots,\,\mathrm{int}(\texttt{G1\_POS}[m])$,

$$S[j-1,k+500] = e^{S[j-1,k+500]}. \tag{104}$$

v. Above the peak of the scatter matrix function:
For $k=\mathrm{int}(\texttt{G1\_POS}[m])+1,\,\mathrm{int}(\texttt{G1\_POS}[m])+2,\,\ldots,\,500$,

$$S[j-1,k+500] = e^{-S[j-1,k+500]}. \tag{105}$$

3. The scatter matrix is normalized:
For $j=1,\,2,\,\ldots,\,4096$:

(a)

$$S_{\mathrm{tot}} = \sum_{k=-500}^{500} S[j-1,k+500]. \tag{106}$$

(b) For $k=-500,\,-499,\,\ldots,\,500$,

$$S[j-1,k+500] = \frac{S[j-1,k+500]}{S_{\mathrm{tot}}}. \tag{107}$$

4.3 Convolve ideal RMF and scatter matrix

To obtain the RMF:

1. The ideal RMF and scatter matrix are convolved:
For $j=1,\,2,\,\ldots,\,4096$,

(a) If

$$j < 501, \tag{108}$$

then

$$k_{\mathrm{lo}} = 1-j. \tag{109}$$

(b) If

$$j \geq 501, \tag{110}$$

then

$$k_{\mathrm{lo}} = -500. \tag{111}$$

(c) If

$$j < 3596, \tag{112}$$

then

$$k_{\rm hi} = 500. \tag{113}$$

(d) If

$$j \geq 3596, \tag{114}$$

then

$$k_{\rm hi} = 4096 - j. \tag{115}$$

(e)

$$R[i, j-1] = \sum_{k=k_{\rm lo}}^{k_{\rm hi}} \mathcal{R}[(j-1)+k]S[j-1, k+500]. \tag{116}$$

5 TBD

1. PI RMFs
2. Nonstandard `channel` ranges or binning
3. Weighting for multiple regions