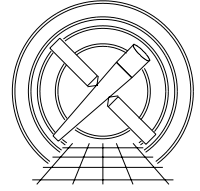


MIT Kavli Institute



Chandra X-Ray Center

MEMORANDUM

June 16, 2020

To: HETGS users
From: Herman L. Marshall
Subject: Effects of ACIS RMF Changes on the HETGS Order Selection
Revision:1
URL: <https://space.mit.edu/CXC/docs/docs.html#hetgfptemp>
File: ~hermannm/CXC/HETGS/Warm ACIS/rmf.effects.tex

1 Introduction

There are three possible impacts on *Chandra* High Energy Transmission Grating Spectrometer (HETGS) observations when using ACIS at off-nominal temperatures. Using HETGS data taken with high ACIS focal plane temperatures, Norbert Schulz has looked at QE changes and Dave Huenemoerder has measured the shift and broadening of the ACIS response function (RMF). Their results are available separately. Here, the quantitative effects on the enclosed power in the order selection are estimated based on a shift of the RMF centroid or broadening of the FWHM.

In the standard analysis of HETGS data, events are selected over a range of energies at each location of the dispersed spectrum based on the expected energy as given by the grating equation.¹ For optimal selection of events relative to background, a lookup table defines the energy limits and provides the correction factor that accounts for events outside of these limits. This table is provided in the Order Sorting, Integrated Probability (OSIP) file in the *Chandra* calibration data base. The integrated probability is usually between 0.90 and 0.98.

¹See the Chandra Proposers Observatory Guide for an overview of the properties of the HETGS and the data provided.

2 Modeling the Integrated Probability

We set up the problem by assuming that any changes of the ACIS focal plane temperature only affects the dominant portion of the ACIS RMF. Furthermore, I assume that the RMF shape fits a Gaussian shape with mean E_0 given by the grating dispersion and variance σ_E^2 . Thus, when the OSIP table includes a fraction f of the integrated probability of the RMF, given by $\phi(E)$, we may compute

$$f = \int_{E_1}^{E_2} \phi(E) dE = \frac{1}{(2\pi)^{1/2} \sigma_E} \int_{E_1}^{E_2} e^{-\frac{(E-E_0)^2}{2\sigma_E^2}} dE. \quad (1)$$

Defining a normalized variable $z = (E - E_0)/\sigma_E$, then the integrated probability of the unperturbed RMF is just the integral of a normalized Gaussian:

$$f = \int_{-a}^a \psi(z) dz = \frac{1}{(2\pi)^{1/2}} \int_{-a}^a e^{-\frac{z^2}{2}} dz = [\text{erf}(a/2^{1/2}) - \text{erf}(-a/2^{1/2})]/2, \quad (2)$$

where we now assume symmetric energy limits: $E_0 - E_1 = E_2 - E_0 = a\sigma_E$ and $\text{erf}(x)$ is the classical error function defined by

$$\text{erf}(x) = \frac{2}{(\pi)^{1/2}} \int_0^x e^{-t^2} dt. \quad (3)$$

Simulations of perturbed RMFs use Eq. 2 with suitable perturbations of the integration limits.

2.1 RMF Width Changes

Fig. 1 shows $f(a)$ (left) and how f changes when the RMF is broadened by 10% (right). Defining $\sigma_E = \sigma_0 + \delta\sigma$, the integrated probability for a perturbed RMF is just shifted by rescaling: $a' = a(1 + \delta\sigma/\sigma_0)$. The value of a is set by the unperturbed case (at nominal, cold ACIS temperature), so the loss of integrated probability is given by determining f at the unperturbed value of a . Using Fig. 1, if $f_0 = 0.95$, then $a = 1.96$ for the unperturbed case but when σ is increased 10%, then f is reduced from 0.95 to about 0.925, a 2.5% loss of integrated probability.

A few cases were computed empirically to determine $\delta f/f_0 = 1 - f/f_0$, shown in Fig. 2. Using Eq. 2, we can compute the effect of broadening the RMF analytically to first order in $\delta a \equiv \Delta = a\delta\sigma/\sigma_0$:

$$\delta f = f - f_0 \quad (4)$$

$$= \int_{-a-\Delta}^{a+\Delta} \psi(z) dz - \int_{-a}^a \psi(z) dz \quad (5)$$

$$= \int_0^{a+\Delta} \psi(z) dz + \int_{-a-\Delta}^0 \psi(z) dz - \int_0^a \psi(z) dz - \int_{-a}^0 \psi(z) dz \quad (6)$$

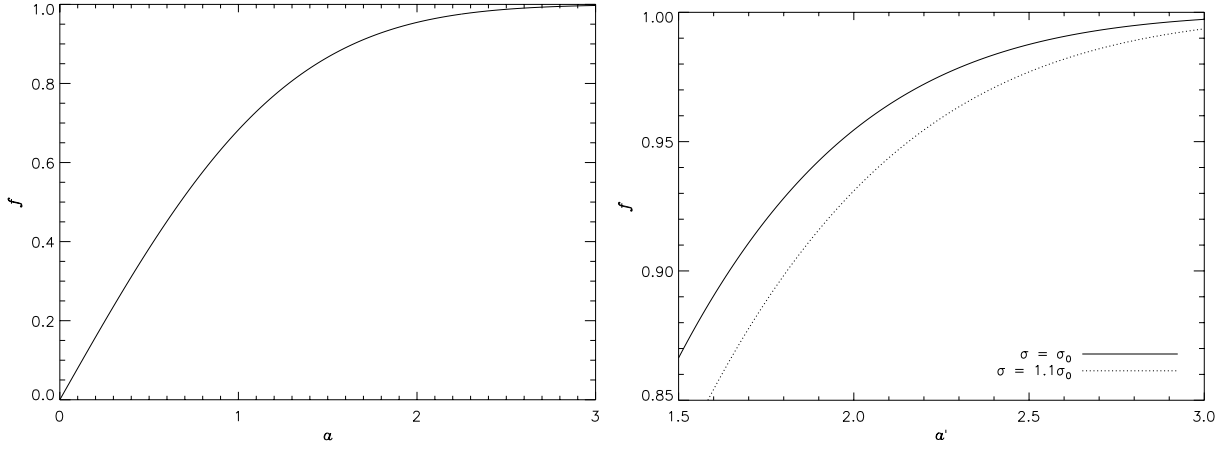


Figure 1: *Left:* The symmetric integrated probability for a Gaussian $f(a)$, given by Eq. 2. For a given value of f_0 , say 0.95, the limits of the integral of an unperturbed Gaussian are set by this curve. *Right:* An example of a perturbed Gaussian, where σ is 10% wider than expected, is shown as the dotted line, compared to the unperturbed version (a solid line). When σ is increased, the portion of the Gaussian to enclose, given by $\pm a$ has to increase to $\pm a'$ in order to enclose the same probability as an unperturbed Gaussian. Otherwise, the enclosed probability is reduced.

$$= 2\left[\int_0^{a+\Delta} \psi(z)dz - \int_0^a \psi(z)dz\right] \quad (7)$$

$$\approx 2\psi(a)\Delta \quad (8)$$

$$= \frac{2\Delta}{(2\pi)^{1/2}} e^{-\frac{a^2}{2}} \quad (9)$$

where we have taken advantage of the symmetric nature of $\psi(z)$. The model is also plotted in Fig. 2, showing that it works well, providing the effect on the integrated probability to better than 10% for $\Delta < 0.1$, so the analytic model can be used to estimate OSIP losses when a fractional RMF broadening is measured.

The results obtained by Dave Huenemoerder show that σ changes by less than 5%, indicating that the integrated probability would change less than 0.5% (1.3%) for an OSIP that integrates 99% (95%) of the Gaussian core of the RMF.

2.2 RMF Centroid Changes

We proceed in a similar fashion as in the last section. For a given location along the dispersion, E_0 is the expected energy but a degradation of the RMF can cause the average ACIS event energy to be $\bar{E} = E_0 + \delta E \equiv (\mu + \delta\mu)\sigma_E$, with $\delta\mu < 0$ most likely. Examples are shown in Fig. 3. As in the last section, empirical fractional changes to f were determined for various shifts of the centroid of the RMF, shown in Fig. 4.

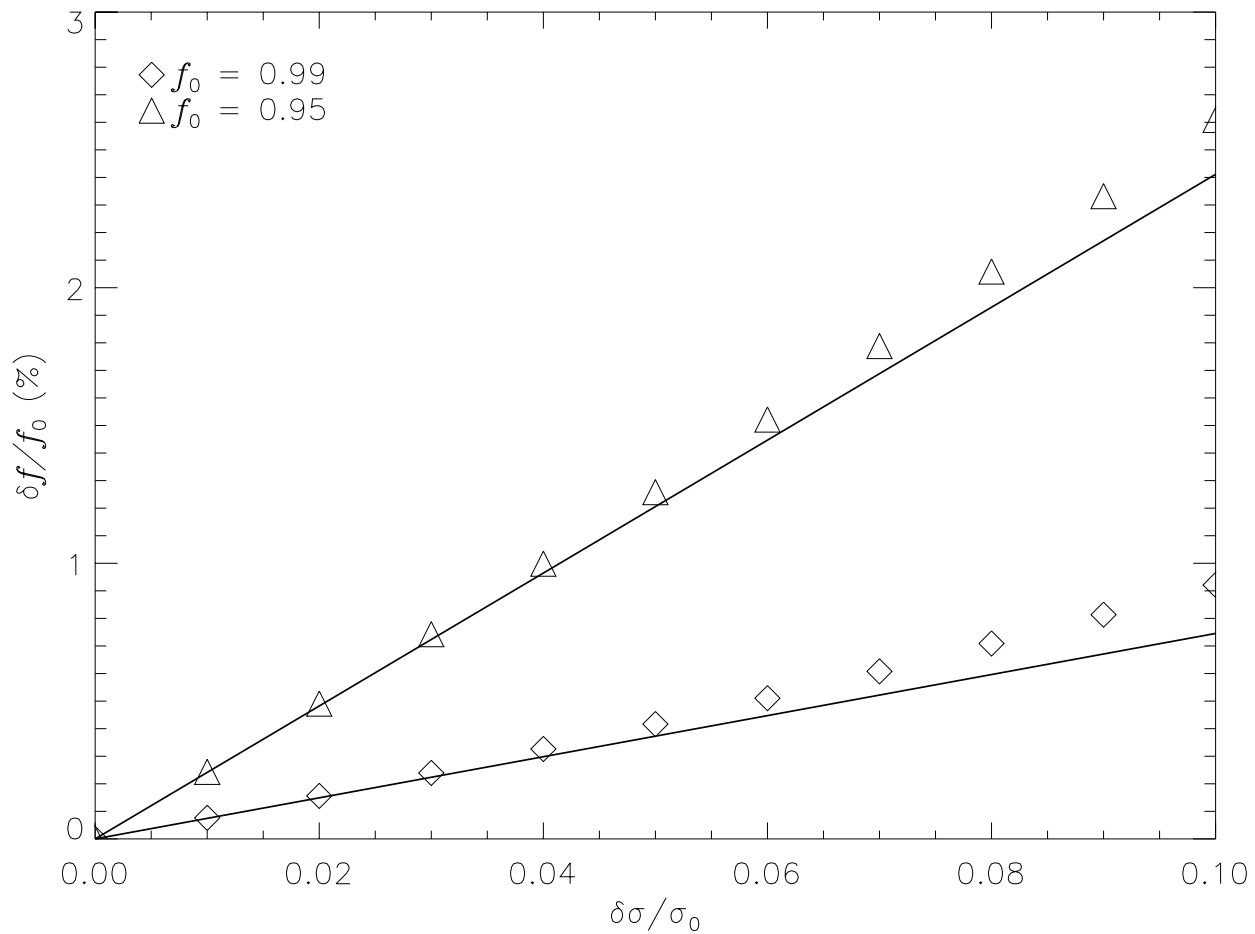


Figure 2: Results from simulating RMF broadening (symbols) compared to the model (thick lines).

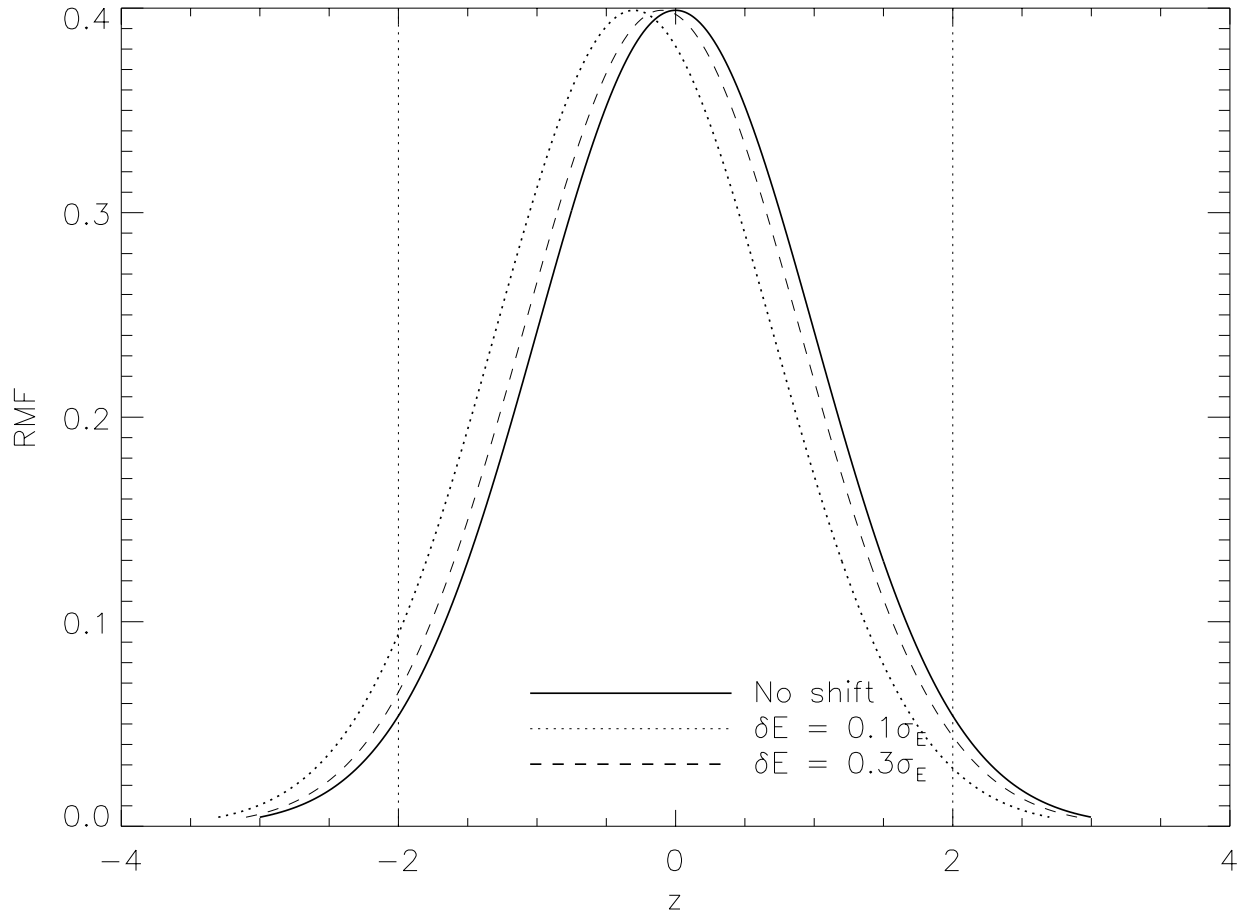


Figure 3: Illustration of how the RMF may shift if the centroid deviates from the value expected based on the HETGS dispersion relation. With shifts to smaller \bar{E} , the loss in the negative tail is larger than the gain in the positive tail.

For the model, we note that this time the change to the limits of the integral have the same signs; both limits increase with $\delta\mu$:

$$\delta f = f - f_0 \tag{10}$$

$$= \int_{-a+\delta\mu}^{a+\delta\mu} \psi(z)dz - \int_{-a}^a \psi(z)dz \tag{11}$$

$$= \int_0^{a+\delta\mu} \psi(z)dz + \int_{-a+\delta\mu}^0 \psi(z)dz - \int_0^a \psi(z)dz - \int_{-a}^0 \psi(z)dz \tag{12}$$

$$= \int_0^{a+\delta\mu} \psi(z)dz - \int_0^a \psi(z)dz + \int_0^{a-\delta\mu} \psi(z)dz - \int_0^a \psi(z)dz \tag{13}$$

$$\approx \psi'(a)(\delta\mu)^2 \tag{14}$$

$$= \frac{-a(\delta\mu)^2}{(2\pi)^{1/2}} e^{-\frac{a^2}{2}} \tag{15}$$

where the approximation yielding Eq. 14 is carried out to second order in $\delta\mu$ because the first order term is zero. Intuitively, the integrated probability should decrease regardless of the sign of $\delta\mu$, so it is reasonable that the first order term is null because it would be linear in $\delta\mu$. Again, the analytic model is a good approximation to the results from the empirical calculations, as seen in Fig. 4.

The results obtained by Dave Huenemoerder show that $\delta E/E_0$ shifts by less than 0.4% for all energies. The values of $\delta E/\sigma_E = \delta\mu$ are 0.1 and 0.3 for E_0 of 1 and 4 keV, respectively; $\delta\mu$ is smaller because σ_E is approximately independent of energy for ACIS while δE decreases with E_0 . For $\delta\mu$ in this range, the effect on the integrated probability is 0.05-0.35% for the more inclusive OSIPs (where $f_0 = 0.99$) but could be as high as 0.8% for an OSIP that includes only 95% of the RMF's Gaussian core.

2.3 Summary

The combined effects of RMF broadening and centroid shifting appear likely to affect the integrated probabilities in the OSIP file by less than 1%, based on results for possible RMF changes found by Dave Huenemoerder. Combined with a null result on QE decrease found by Norbert Schulz (with a limit of < 3%, we conclude that the HETGS data will be minimally impacted if the ACIS focal plane temperature is allowed to rise to -109C.

The formalism developed here can be applied to results of further analysis that refines the effects of higher temperature on the ACIS RMF.

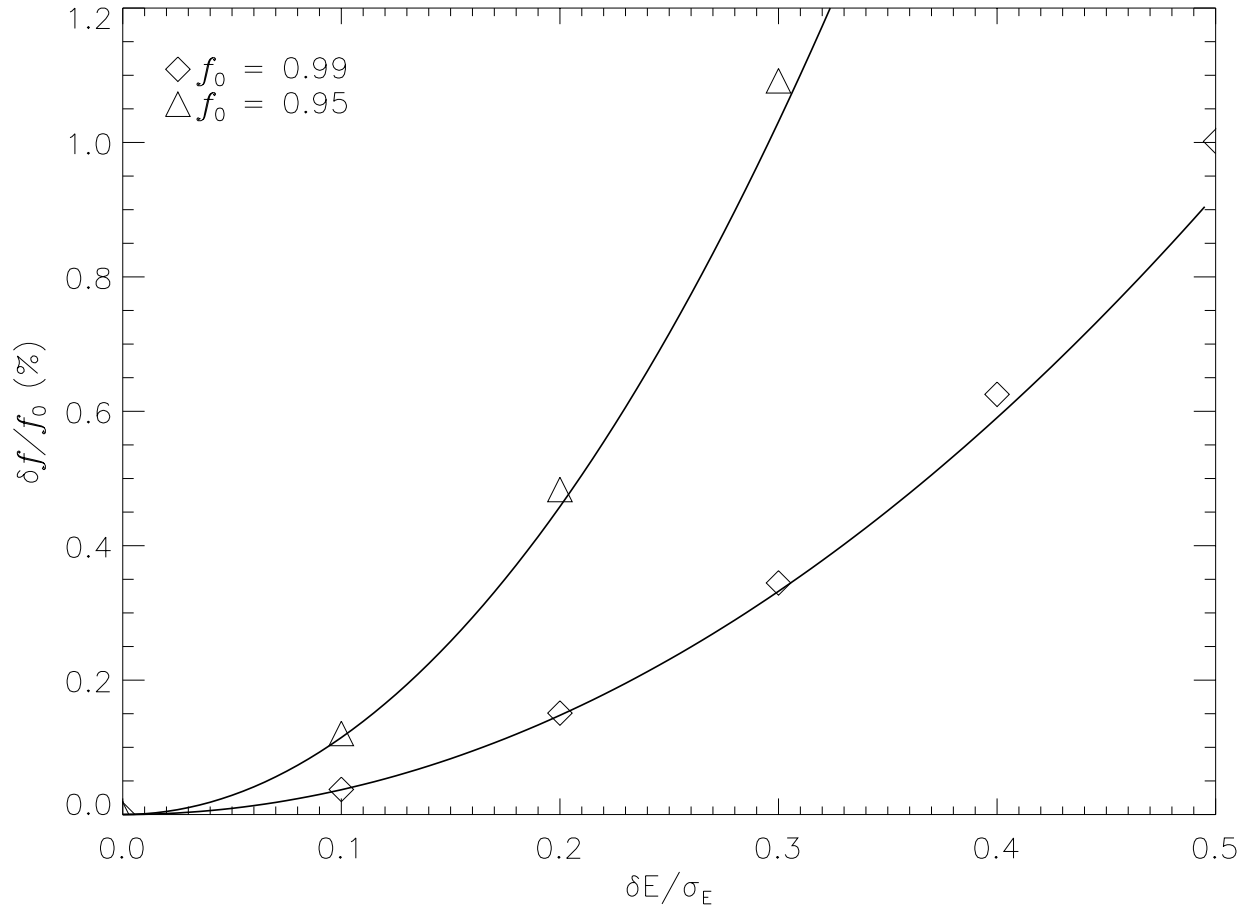


Figure 4: Results from simulating RMF shifts (symbols) compared to the model (thick lines).