Exposure Maps

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1 Introduction

The purpose of this memo is two-fold: to provide a formal definition of an exposure map, and to describe their computation. I have tried to keep the discussion as general as possible, and not be too AXAF specific, in the hope that the definitions presented here will apply to future missions. The defintion of the exposure map was arrived at by noting what one intends to do with such objects and then demanding that the resulting definition be consistent with use, intuition, and above all, physics.

2 Notation and Conventions

The mathematical notation used in this memo is typical of what is used in most physics texts. Specifically,

- 3-vectors will be written using with an arrow on top, e.g., $\vec{a}$.
- a unit 3-vector will be written with a "hat", e.g., $\hat{p}$, and, unless otherwise stated, such vectors will represent represent *directions* or *rays*. With this understanding, $\hat{p}$ should be thought of as a pair of angles (θ, ϕ), and the infinitesimal unit vector $\mathrm{d}\hat{p}$ will specify an infinitesimal solid angle about the direction $\hat{p}$.
- The symbol σ will denote a position on the detector and $\mathrm{d}\sigma$ represents the area of an infinitesmal region on the detector. Thus σ represents two scalars, e.g., the x and y pixel values.

- rotation matrices will be represented with script characters, e.g., $\mathcal{R}$.

3 Telescope Response

Let $S(\lambda, \hat{p})\mathrm{d}\hat{p}\,\mathrm{d}\lambda$ represent the number of photons incident upon the telescope, per unit area, per unit time, and whose directions lie in the cone of directions between $\hat{p}$ and $\hat{p} + \mathrm{d}\hat{p}$, and whose wavelengths lie between λ and $\lambda + d\lambda$. The source flux shall be assumed to be time-independent. Similarly, let $S_D(\sigma, h, t)\mathrm{d}\sigma$ represent the number of counts per unit time, with pulse-height h within a region of area $\mathrm{d}\sigma$ at the position σ on the detector.

In general, the count rate will be time-dependent even if the source flux does not vary with time. This time-dependence arises from several effects, including

- spacecraft dither.
- detector good-time intervals.
- movement of individual telescope subsystems with respect to one another and caused by e.g., thermal expansion.

As used here, dither is the movement of a coordinate system attached to the optical axis with respect to a coordinate system fixed with respect to the sky. The two coordinate systems are related by a time-dependent rotation matrix $\mathcal{R}(t)$ which completely characterizes the dither. Without any loss of generality, it will be assumed that the two coordinate systems are aligned at $t = 0$, i.e., $\mathcal{R}(0) = 1$.

In the coordinate system fixed with respect to optical axis, the source flux will appear to be time-dependent according to

$$S(\lambda, \hat{p}, t) = S(\lambda, \mathcal{R}^{-1}(t) \cdot \hat{p}). \tag{1}$$

In this coordinate system, the source flux and the count rate are related to one another via the equation

$$S_D(\sigma, h, t) = \int \mathrm{d}\lambda \int \mathrm{d}\hat{p}\; R_T(\sigma, h, \lambda, \hat{p}, t) S(\lambda, \mathcal{R}^{-1}(t) \cdot \hat{p}), \tag{2}$$

which defines the *response* R_T of the telescope. R_T is an extremely complicated function that incorporates all elements of the telescope such as the detector and its electronics, the mirror, and the gratings. Since the units of S_D are counts per unit area per unit time, and the units of $S\mathrm{d}\lambda$ are photons per unit area per unit time, we infer that the response R_T is a unitless quantity (counts per photon).

The explicit time-dependence of R_T is due to the detector electronics (good-time intervals) and thermal expansion effects. We shall assume that the time-dependence associated with the electronics factors out of R_T:

$$R_T(\sigma, h, \lambda, \hat{p}, t) = T(\sigma, t) R(\sigma, h, \lambda, \hat{p}, t) \tag{3}$$

In this equation the explicit time-dependence of the response function R depends only upon the relative movements of the individual telescope subsystems (thermal effects) and not upon the good-time intervals. The time-dependence of the good-time intervals are incorporated in function T, which may also depend upon individual detector regions since an arbitrary region of the detector may have region-dependent good-time interval associated with it. The role of T can be also be expanded to include bad-pixel lists, which themselves may be time-dependent. Thus, (2) can be written

$$\begin{aligned} S_D(\sigma, h, t) &= T(\sigma, t) \int \mathrm{d}\lambda \int \mathrm{d}\hat{p}\; R(\sigma, h, \lambda, \hat{p}, t) S(\lambda, \mathcal{R}^{-1}(t) \cdot \hat{p}) \\ &= T(\sigma, t) \int \mathrm{d}\lambda \int \mathrm{d}\hat{p}\; R(\sigma, h, \lambda, \mathcal{R}(t) \cdot \hat{p}, t) S(\lambda, \hat{p}) \end{aligned} \tag{4}$$

where the last form follows from the change of variables $\hat{p} \longrightarrow \mathcal{R}(t) \cdot \hat{p}$ and noting that the Jacobian of an orthogonal matrix is one.

Since the response depends upon several *independent* telescope subsystems, we may factor it into subsystem dependent pieces, e.g.,

$$R(\sigma, h, \lambda, \hat{p}, t) = D(\sigma, h, \lambda) G(\sigma, \lambda, \hat{p}, t) M(\lambda, \hat{p}), \tag{5}$$

where $M(\lambda, \hat{p})$ is the off-axis effective area of the mirror, and $D(\sigma, h, \lambda)$ represents the probability that a photon of wavelength λ at position σ on the detector will give rise to pulse-height h. Finally, $G(\sigma, \lambda, \hat{p}, t)\mathrm{d}\sigma$ is the probability that a photon of wavelength λ and direction $\hat{p}$ will land within an area of size $\mathrm{d}\sigma$ centered at position σ of the detector. Hence, $D(\sigma, h, \lambda)$ is a unitless quantity, whereas $M(\lambda, \hat{p})$ has the units of area and $G(\sigma, \lambda, \hat{p}, t)$ has the units of inverse area.

The function $G(\sigma, \lambda, \hat{p}, t)$ could represent the PSF of the mirror, or it could represent the dispersion of photons by a diffraction grating. For the imaging case (no gratings) and a perfect PSF, we can write

$$G(\sigma, \lambda, \hat{p}, t) = \delta(\sigma - \sigma(\hat{p}, t)) \tag{6}$$

where $\sigma(\hat{p}, t)$ is a function that represents the (time-dependent) coordinate transformation from the direction $\hat{p}$ to the detector coordinate σ.

When gratings are present, and assuming no grating scatter, we can write

$$G(\sigma, \lambda, \hat{p}, t) = \sum_{n=-\infty}^{\infty} G_n(\sigma, \lambda, \hat{p}, t) \tag{7}$$

where

$$G_n(\sigma, \lambda, \hat{p}, t) = g_n(\lambda)\delta(\sigma - \sigma_n(\lambda, \hat{p}, t)). \tag{8}$$

Here, $g_n(\lambda)$ represents the grating efficiency for the n-th order, and the function $\sigma_n(\lambda, \hat{p}, t)$ represents the coordinate transformation for a photon with direction $\hat{p}$ and wavelength λ going into the n-th order at position σ on the detector.

4 Exposure Map

The exposure map, $E(h, \lambda, \hat{q})$ is defined

$$E(h, \lambda, \hat{p}) = \int \mathrm{d}\sigma \int_0^\tau \mathrm{d}t\, T(\sigma, t) R(\sigma, h, \lambda, \hat{p}_t, t), \tag{9}$$

where τ is the observation time. Here the time dependent quantity

$$\hat{p}_t = \mathcal{R}(t) \cdot \hat{p}. \tag{10}$$

has been introduced to simplify the notation. The exposure map has units of cm^2-seconds and represents the total effective area given by the telescope to a point $\hat{p}$ in the sky at wavelength λ and giving rise to a pulse-height h.

From (9) one can readily show that

$$\int_0^\tau \mathrm{d}t \int \mathrm{d}\sigma\, S_D(\sigma, h, t) = \int \mathrm{d}\lambda \int \mathrm{d}\hat{p}\, E(h, \lambda, \hat{p}) S(\lambda, \hat{p}), \tag{11}$$

which is an integral equation that relates the total number of counts with pulse-height h over the course of the observation to the source flux.

4.1 Imaging Mode

If one assumes a perfect mirror PSF, then knowing the instantaneous pointing of the spacecraft (aspect solution), it is possible to determine where a given photon came from in the sky. That is, one can define *aspect-corrected* data via the expression

$$S_A(h, \hat{q}, t) = J(\sigma(\hat{q}_t, t), \hat{q}, t) S_D(h, \sigma(\hat{q}_t, t), t) \tag{12}$$

where $J(\sigma, \hat{q}, t)$ is the instantaneous Jacobian of the transformation from the detector coordinate σ to the sky coordinate $\hat{q}$. Physically, the Jacobian represents the stretching or magnification of an element of area on the detector as it appears in the sky.

At the same level of approximation, we write the response as

$$R(\sigma, h, \lambda, \hat{p}_t, t) = D(\sigma, h, \lambda)\delta(\sigma - \sigma(\hat{p}_t, t))M(\lambda, \hat{p}_t), \tag{13}$$

which upon substitution into (9) yields

$$E_I(h, \lambda, \hat{p}) = \int_0^\tau \mathrm{d}t\; T(\sigma(\hat{p}_t, t), t)D(\sigma(\hat{p}_t, t), h, \lambda)M(\lambda, \hat{p}_t). \tag{14}$$

This equation represents the exposure map for the imaging case.[1]

Upon substitution of (4) into (12) we obtain after integrating over the observation time

$$\int_0^\tau \mathrm{d}t\; S_A(h, \hat{q}, t) = \int \mathrm{d}\lambda\; E_I(h, \lambda, \hat{q})S(\lambda, \hat{q}). \tag{15}$$

In deriving this equation, it was necessary to make use of the result

$$\delta(\sigma(\hat{p}_t, t) - \sigma(\hat{q}_t, t)) = \frac{\delta(\hat{p} - \hat{q})}{J(\sigma(\hat{q}_t, t), \hat{q}, t)} \tag{16}$$

in order to perform the integration over $\hat{p}$.

Equation (15) is an integral equation that relates the expected number of counts over the course of an observation with pulse-height h to a source at position $\hat{q}$ in the sky. Given the exposure map $E_I(h, \lambda, \hat{q})$ and the aspect-corrected data, one would like to invert this equation for $S(\lambda, \hat{q})$. In general this is not possible without using some sort of forward folding technique, e.g., as implemented by `xspec`, or without making some assumption about the nature of the source spectrum.

4.2 Spectroscopic Mode

For the case where the diffraction gratings are used, we shall assume that the response function can be written as

$$R(\sigma, h, \lambda, \hat{p}_t, t) = \sum_{n=-\infty}^{\infty} R_n(\sigma, h, \lambda, \hat{p}, t), \tag{17}$$

[1] The subscript I is used to indicate that E_I is an imaging exposure map and will serve to distinguish it from the spectroscopic exposure map E_S.

where

$$R_n(\sigma, h, \lambda, \hat{p}_t, t) = D(\sigma, h, \lambda) M(\lambda, \hat{p}_t) g_n(\lambda) \delta(\sigma - \sigma_n(\lambda, \hat{p}_t, t)) \tag{18}$$

represents the response of the telescope for photons going into the nth diffraction order. In this equation $\sigma_n(\lambda, \hat{p}, t)$ specifies the coordinate transformation associated with the diffraction of a photon with direction $\hat{p}$ and wavelength λ to the detector coordinate σ, and $g_n(\lambda)$ is the grating efficiency for the nth order.

Inserting the response into (9) yields the spectroscopic exposure map

$$E_S(h, \lambda, \hat{p}) = \sum_{n=-\infty}^{\infty} E_S^{(n)}(h, \lambda, \hat{p}), \tag{19}$$

where the exposure map for n-th order events is given by

$$E_S^{(n)}(h, \lambda, \hat{p}) = g_n(\lambda) \int_0^\tau \mathrm{d}t \, T(\sigma_n(\lambda, \hat{p}_t, t), t) D(\sigma_n(\lambda, \hat{p}_t, t), h, \lambda) M(\lambda, \hat{p}_t). \tag{20}$$

Despite the apparent simplicity of this equation and the formal resemblance to imaging exposure map given in (14), there is one extremely important difference between the two: the coordinate transformation $\sigma_n(\lambda, \hat{p}_t, t)$ depends upon the wavelength whereas the analogous transformation in (14) does not. Nevertheless, the imaging exposure map is related to the zeroth order spectroscopic exposure map via

$$E_S^{(0)}(h, \lambda, \hat{p}) = g_0(\lambda) E_I(h, \lambda, \hat{p}). \tag{21}$$

If one is dealing with a point source and using a detector with sufficient energy resolution to permit order sorting (ACIS-S), a more useful definition for an exposure map can be given. For such a detector, one can apply a filter $F_n(\sigma, h, \hat{p}_t)$ to the measured count rate and separate out events from the nth order, i.e.,

$$S_D^{(n)}(\sigma, h, t; \hat{p}_t) = F_n(\sigma, h, \hat{p}_t) S_D(\sigma, h, t) \tag{22}$$

F_n simply represents the correlation between the nth order events occuring at the position σ on the detector and the pulse heights h of those events, for a specified zeroth order location determined by $\hat{p}_t$.

Using (4) it trivially follows that

$$\begin{aligned} S_D^{(n)}(\sigma, h, t; \hat{p}_t) = {} & T(\sigma, t) F_n(\sigma, h, \hat{p}_t) \\ & \times \int d\lambda \, S(\lambda) g_n(\lambda) D(\sigma, h, \lambda) M(\lambda, \hat{p}_t) \delta(\sigma - \sigma_n(\lambda, \hat{p}_t, t)). \end{aligned} \tag{23}$$

Now define the function

$$\Delta_\epsilon(\sigma;\lambda_G,\hat{p}_t,t) = \begin{cases} 1 & \text{if } |\sigma - \sigma_n(\lambda_G,\hat{p}_t,t)| < \epsilon \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

This function is non-zero only on a region of the detector of radius ϵ centered at the point given by $\sigma_n(\lambda_G,\hat{p}_t,t)$. It will be used as a filter to select events whose wavelengths are near the value λ_G.

Applying this filter to (23) and integrating over σ yields

$$\begin{aligned} &\int \mathrm{d}\sigma\; S_D^{(n)}(\sigma,h,t)\Delta_\epsilon(\sigma;\lambda_G,\hat{p}_t,t) \\ &\qquad = \int d\lambda\; S(\lambda)\Big\{T(\sigma_n,t)F_n(\sigma_n,h,\hat{p}_t)\Delta_\epsilon(\sigma_n;\lambda_G,\hat{p}_t,t) \\ &\qquad\qquad \times g_n(\lambda)D(\sigma_n,h,\lambda)M(\lambda,\hat{p}_t)\Big\} \end{aligned} \tag{25}$$

where for notational simplicity, σ_n is really $\sigma_n(\lambda,\hat{p}_t,t)$. If ϵ is small, e.g., on the order of the size of a pixel, only a narrow band of wavelengths λ concentrated near λ_G will contribute to the integral. One should note that the size $\Delta\lambda$ of this wavelength band depends upon time, as well as λ_G and $\hat{p}_t$. From these considerations, it follows that

$$\begin{aligned} &\int \mathrm{d}\sigma\; S_D^{(n)}(\sigma,h,t)\Delta_\epsilon(\sigma;\lambda_G,\hat{p}_t,t) \\ &\quad \approx S(\lambda_G)\Delta\lambda\Big\{T(\sigma_n,t)F_n(\sigma_n,h,\hat{p}_t)g_n(\lambda_G)D(\sigma_n,h,\lambda_G)M(\lambda_G,\hat{p}_t)\Big\}. \end{aligned} \tag{26}$$

Finally, after an integration over time we arrive at

$$S(\lambda_G)E_G^{(n)}(h,\lambda_G,\hat{p}) \approx \int_0^\tau \mathrm{d}t \int \mathrm{d}\sigma\; S_D^{(n)}(\sigma,h,t)\Delta_\epsilon(\sigma;\lambda_G,\hat{p}_t,t), \tag{27}$$

where

$$\begin{aligned} &E_G^{(n)}(h,\lambda_G,\hat{p}) \\ &\quad = g_n(\lambda_G)\int_0^\tau \mathrm{d}t\; \Delta\lambda\Big\{T(\sigma_n,t)F_n(\sigma_n,h,\hat{p}_t)D(\sigma_n,h,\lambda_G)M(\lambda_G,\hat{p}_t)\Big\}, \end{aligned} \tag{28}$$

and σ_n is shorthand for $\sigma_n(\lambda_G,\hat{p}_t,t)$.

A few comments about this equation are in order:

1. As noted above, $\Delta\lambda$ is a time-dependent quantity that depends upon the size of the extraction region. It is conceivable that if the extraction region

is small enough and the dither pattern is small, that $\Delta\lambda$ is effectively constant for all wavelength ranges of interest. This would allow $\Delta\lambda$ to be removed from the integrand.

2. $E_G(h, \lambda_G, \hat{p})$ depends upon the filter that was used to perform the order extraction. If the amplitude of the dither is small on the scale of any detector non-uniformities, then F_n can be regarded as a constant and removed from the integrand. However, this is possible only when the dither does not cause the nth order position to wander between two regions of the detector that differ in their response. For example, if the point moves between front side and back side chips, then F_n cannot be removed from the integrand. For front-side illuminated chips, the detector response is expected to be uniform across the CCD.

Assuming a front-side CCD and small dither, we may sum over pulse-heights to yield

$$\sum_h E_G^{(n)}(h, \lambda_G, \hat{p}) = \Delta\lambda\, g_n(\lambda_G) \int_0^\tau \mathrm{d}t\, T(\sigma_n, t) M(\lambda_G, \hat{p}_t) \sum_{h=h_0(\lambda_G)}^{h_1(\lambda_G)} D(\sigma_n, h, \lambda_G), \quad (29)$$

where the functions $h_0(\lambda_G)$ and $h_1(\lambda_G)$ specifiy the minimum and maximum pulse heights appropriate for the wavelength λ_G.

5 Computation of the Exposure Map

In principle the exposure map is a 4 dimensional object that depends upon pulse height, wavelength, and the two coordinates that define the position on the sky. A high resolution expsoure map for a detector with 4096 pulse height channels, defined on a sky 1024 by 1024 sky grid at 100 energies would require over 1.6 terabytes of space (assuming 4 bytes per cell)! Clearly it is not feasible to compute such an object in all its glory. Rather, it will have to be computed at lower resolution or in various wavelength or pulse height bands.

5.1 Imaging Case

The imaging exposure map is defined by (14) in section 4.1. For convenience, it is repeated below:

$$E_I(h, \lambda, \hat{p}) = \int_0^\tau \mathrm{d}t\, T(\sigma(\hat{p}_t, t), t) D(\sigma(\hat{p}_t, t), h, \lambda) M(\lambda, \hat{p}_t). \quad (30)$$

Generally speaking, the detector response $D(\sigma, h, \lambda)$ can be factored into a quantum efficiency (QE) and a redistribution (RMF), i.e.,

$$D(\sigma, h, \lambda) = D_R(\sigma, h, \lambda) Q(\sigma, \lambda). \tag{31}$$

where the redistribution function $D_R(\sigma, h, \lambda)$ is normalized via

$$1 = \sum_h D_R(\sigma, h, \lambda). \tag{32}$$

Alternatively, the QE may be defined by

$$Q(\sigma, \lambda) = \sum_h D(\sigma, h, \lambda). \tag{33}$$

For detectors with poor intrinsic energy resolution (HRC), it makes sense to reduce the dimensionality of the exposure map by summing over all pulse height channels. However, doing so for an instrument with good pulse height resolution is undesirable since we would then lose the information provided by the pulse heights. Instead, we will allow for the possibility of summing over a specified range of pulse heights Δh by defining

$$D(\sigma, \Delta h, \lambda) = \sum_{h \in \Delta h} D(\sigma, h, \lambda) \tag{34}$$

where the notation $D(\sigma, \Delta h, \lambda)$ indicates that we have summed over the pulse height range Δh. Thus we define

$$E_I(\Delta h, \lambda, \hat{p}) = \int_0^\tau \mathrm{d}t\, T(\sigma(\hat{p}_t, t), t) D(\Delta h, \sigma(\hat{p}_t, t), \lambda) M(\lambda, \hat{p}_t), \tag{35}$$

as the exposure map over the pulse height range Δh.

The explicit time dependence of the coordinate transformation $\sigma(\hat{p}_t, t)$ is due to thermal stresses, which are believed to be quite small. Their influence upon the coordinate transformation should be no more than a few pixels, across which the variation of the mirror effective area $M(\lambda, \hat{p}_t)$ can be ignored. Thus, we can make a change of variables to project $M(\lambda, \hat{p}_t)$ onto the detector coordinate system and disregard any error introduced in this process by the motion of the detector induced by thermal effects. That is, we may regard the mirror effective area as a function of σ via the change of variables defined by the equation

$$\sigma = \sigma(\hat{p}, t_0) \tag{36}$$

where t_0 can arbitrarily be chosen to be zero. Hence, in the following, $M(\sigma, \lambda)$ shall denote the mirror effective area in this coordinate system. With this understanding we may approximate (35) as

$$E_I(\Delta h, \lambda, \hat{p}) = \int_0^\tau \mathrm{d}t\, T(\sigma(\hat{p}_t, t), t) D(\Delta h, \sigma(\hat{p}_t, t), \lambda) M(\sigma(\hat{p}_t, t), \lambda). \tag{37}$$

This expression can be simplified by defining an *Instrument Map* $I(\Delta h, \lambda, \sigma)$ as the product of the QE and the effective area, i.e.,

$$I(\sigma, \Delta h, \lambda) = D(\Delta h, \sigma, \lambda) M(\sigma, \lambda). \tag{38}$$

Then, the expression for the exposure map becomes

$$E_I(\Delta h, \lambda, \hat{p}) = \int_0^\tau \mathrm{d}t\, T(\sigma(\hat{p}_t, t), t) I(\sigma(\hat{p}_t, t), \lambda). \tag{39}$$

If one can assume that the source does not vary much over some wavelength band, then the instrument map can be integrated over the band band to define an instrument map that is suitable for that range of wavelengths:

$$I(\sigma, \Delta h, \Delta\lambda) = \frac{1}{\Delta\lambda} \int_{\lambda \in \Delta\lambda} \mathrm{d}\lambda\, I(\sigma, \Delta h, \lambda), \tag{40}$$

i.e., $I(\sigma, \Delta h, \Delta\lambda)$ represents the average value of the instrument map over the wavelength band $\Delta\lambda$. Similarly, we define

$$\begin{aligned} E_I(\Delta h, \Delta\lambda, \hat{p}) &= \frac{1}{\Delta\lambda} \int_{\lambda \in \Delta\lambda} \mathrm{d}\lambda\, E_I(\Delta h, \Delta\lambda, \hat{p}) \\ &= \int_0^\tau \mathrm{d}t\, T(\sigma(\hat{p}_t, t), t) I(\sigma(\hat{p}_t, t), \Delta h \Delta\lambda) \end{aligned} \tag{41}$$

as the exposure map appropriate for the wavelength band $\Delta\lambda$ and pulse height range Δh.

Thus, the computation (41) requires the following information:

- An instrument map apppropriate for some wavelength band and a range of pulse heights. The map can include such items as dead pixels, detector windows, filter boundaries, etc.
- The good-time intervals, $T(\sigma, t)$. This value of this function will most likely be zero or one, depending upon whether or not the part of the detector specified by σ is 'on' or 'off'. If this time-dependence does not vary within the the region of the detector that is appropriate for the exposure map, then it will reduce to a one-demensional function. In fact, if T can be regarded as either 'on' or 'off', one only needs to store the start and stop time intervals, i.e., as in a GTI fits file.
- The aspect solution. These quantities are implicit in the various coordinate transformations that appear throughout this document. For AXAF, the aspect solution is six dimensional and the geometric state of the telescope can be represented as a point in this six dimensional space. In particular, three of the dimensions (RA, Dec, ROLL) correspond to the

orientation of the optical axis and the roll of the spacecraft about the optical axis. The other three dimensions specify the location of the SIM with repect to its nominal position $(\mathrm{d}Y, \mathrm{d}Z, \mathrm{d}\theta)$. It will be important to compress the aspect history as much as possible since the time to compute the exposure map varies linearly with the number or aspect points.

6 Uses of the Exposure Map

In this section we outline several potential uses of the exposure map and we pay particular attention to the validity of such uses.

For an imaging observation, the expsoure map is related to the aspect corrected count rate S_A via (15), which we shall write in the form

$$C(h, \hat{p}) = \int \mathrm{d}\lambda \; E_I(h, \lambda, \hat{p}) S(\lambda, \hat{p}), \tag{42}$$

where the total number of counts at sky coordinate $\hat{p}$ is given by

$$C(h, \hat{p}) = \int_0^\tau \mathrm{d}t \; S_A(h, \hat{p}, t). \tag{43}$$

It cannot be emphasized enough that the (42) is an integral equation and that one cannot simply divide by the exposure map E_I without making some assumptions about the energy dependence of the source spectrum S.

In general E_I will vary with energy, with the most abrupt variations near the edges of iridium, silicon, etc. However, away from these edges, E_I may be regarded as a constant across narrow enough wavelength bands. In fact, as pointed out in the previous section, it is not feasable to compute E_I at every wavelength, rather its average value will be computed over various wavelength bands.

Summing (42) over the pulse height range Δh and making use of (35) yields

$$C(\Delta h, \hat{p}) = \int \mathrm{d}\lambda \; E_I(\Delta h, \lambda, \hat{p}) S(\lambda, \hat{p}), \tag{44}$$

Now break up the integration over λ into wavelength bands of width $\Delta\lambda_i$ to produce

$$C(\Delta h, \hat{p}) = \sum_i \int_{\Delta\lambda_i} \mathrm{d}\lambda \; E_I(\Delta h, \lambda, \hat{p}) S(\lambda, \hat{p}). \tag{45}$$

If the bands have been chosen such that $E_I(\Delta h, \lambda, \hat{p})$ does not vary much over the band, then we may replace $E_I(\Delta h, \lambda, \hat{p})$ by its average value over the band to yield

$$C(\Delta h, \hat{p}) = \sum_i E_I(\Delta h, \Delta\lambda_i, \hat{p}) \int_{\Delta\lambda_i} \mathrm{d}\lambda \; S(\lambda, \hat{p}). \tag{46}$$

Similarly, we may write define

$$S(\Delta\lambda_i, \hat{p}) = \frac{1}{\Delta\lambda_i} \int_{\Delta\lambda_i} \mathrm{d}\lambda \; S(\lambda, \hat{p}) \tag{47}$$

as the average value of source flux over the wavelength range $\Delta\lambda_i$, and substiute this expression into (46) to yield

$$C(\Delta h, \hat{p}) = \sum_i E_I(\Delta h, \Delta\lambda, \hat{p}) \; S(\Delta\lambda_i, \hat{p}) \; \Delta\lambda_i \tag{48}$$

7 Required Tools

name: `expmap_arf`

description: This program computes the ARF function defined by (**??**).

inputs: Good time intervals
Instrument map
Aspect history (offsets from nominal)
Sky Grid (defined from nominal)
Wavelength Grid

outputs: An fits image defined on the sky grid.

comments: See section 5.