AXAF Response, Aspect Histogram, and Exposure Map:

Summary of Results

Work in a coordinate system where the spacecraft is stationary and the dither appears as a time-dependent source. Then we can write

$$S'(\lambda_D, \hat{p}, t) = \int \mathrm{d}\lambda \int \mathrm{d}\hat{p}'\; R(\lambda_D, \lambda, \hat{p}, \hat{p}', t) S(\lambda, \hat{p}', t) \tag{1}$$

and

$$S(\lambda, \hat{p}', t) = S(\lambda, \mathcal{R}(t) \cdot \hat{p}'), \tag{2}$$

where $\mathcal{R}(t)$ is the rotation matrix that represents the dither. Here $R(\lambda_D, \lambda, \hat{p}, \hat{p}', t)$ represents the total AXAF response. The response factors into mirror, grating, and detector responses:

$$R(\lambda_D, \lambda, \hat{p}, \hat{p}', t) = D(\lambda_D, \lambda, \hat{p}, t) \times \sum_{m,s} \int \mathrm{d}\hat{p}''\; G_m(\lambda, \hat{p}, \hat{p}'') M_s(\lambda, \hat{p}'', \hat{p}'). \tag{3}$$

For a perfect mirror and perfect gratings:

$$M_s(\lambda, \hat{p}'', \hat{p}') = M_s(\lambda, \hat{p}') \delta(\hat{p}'' - \hat{p}'), \tag{4}$$

$$G_m(\lambda, \hat{p}, \hat{p}') = g_m(\lambda) \delta(\hat{p}' - \hat{P}(\hat{p}, m, \lambda)) \tag{5}$$

where the vector valued function $\hat{P}$ satisfies:

$$\hat{P} \times \hat{n} = \hat{p} \times \hat{n} - \frac{m\lambda}{d} \hat{l}. \tag{6}$$

Assume that time-dependence of R factors out:

$$R(\lambda_D, \lambda, \hat{p}, \hat{p}', t) = R(\lambda_D, \lambda, \hat{p}, \hat{p}') T(\hat{p}, t) \tag{7}$$

Integrating over the observation time produces:

$$\begin{aligned} C(\lambda_D, \hat{p}) &= \int \mathrm{d}t\; S(\lambda_D, \hat{p}, t) \\ &= \int \mathrm{d}\lambda \int \mathrm{d}\hat{q}' \int \mathrm{d}\hat{p}' R(\lambda_D, \lambda, \hat{p}, \hat{p}') A(\hat{p}', \hat{q}'; \hat{p}) S(\lambda, \hat{q}') \end{aligned} \tag{8}$$

where A is the **aspect histogram** given by

$$A(\hat{p}', \hat{q}'; \hat{p}) = \int \mathrm{d}t\; T(\hat{p}, t) \delta(\hat{q}' - \mathcal{R}(t) \cdot \hat{p}'). \tag{9}$$

By assuming perfect mirrors and perfect gratings, and a point-source at off-axis position $\hat{q}$,

$$S(\lambda, \hat{q}') = S(\lambda)\delta(\hat{q}' - \hat{q}). \tag{10}$$

we can write

$$C(\lambda_D, \hat{p}) = \sum_{m,s} \int \mathrm{d}\lambda \; D(\lambda_D, \lambda, \hat{p}) g_m(\lambda) M_s(\lambda, \hat{P}(\hat{p}, m, \lambda)) A(\hat{P}(\hat{p}, m, \lambda), \hat{q}; \hat{p}) S(\lambda). \tag{11}$$

For imaging case where no gratings are present, the aspect factors out:

$$C(\lambda_D, \hat{p}) = A(\hat{p}, \hat{q}; \hat{p}) \times \int \mathrm{d}\lambda \; D(\lambda_D, \lambda, \hat{p}) M(\lambda, \hat{p}) S(\lambda). \tag{12}$$

For a monochromatic point-source of wavelength λ and unit-flux given by,

$$S(\lambda') = \delta(\lambda' - \lambda), \tag{13}$$

we deduce that the *total* number of counts it produces over the observation time to be

$$\begin{aligned} E(\lambda_D, \lambda, \hat{q}) &\equiv \int \mathrm{d}\hat{p} \; C(\lambda_D, \hat{p}) \\ &= \int \mathrm{d}\hat{p} \; D(\lambda_D, \lambda, \hat{p}) M(\lambda, \hat{p}) A(\hat{p}, \hat{q}; \hat{p}) \end{aligned} \tag{14}$$

This expression defines the **exposure map** for the imaging case.