AXAF Exposure Maps (version 1.0)

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1 Introduction

This document defines the various "maps" associated with flight and XRCF data processing. A map is an image in some world coordinate system which integrates a properties of the instruments over time at each world coordinate. The detector is generally in motion relative to the world frame during the exposure, either intentionally as with spacecraft or detector dither, or unintentionally, as in pointing instability. The following deals with the known motion whose time dependence is called the "aspect history", which is in turn determined by the "aspect solution". World coordinate maps are built from maps in device local coordinates, such as a quantum efficiency (QE) map in local detector pixel coordinates.

The ultimate purpose of the maps are to form an effective area map and exposure time map which can be used to convert detected counts to flux (photons $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{\AA}^{-1}$) entering the telescope.

Please observe that the exposure map *does not include the point-spread-function!*. The definition of effective area here is integrated to infinity in the point-spread-function (PSF). Application of a PSF region and an exposure map to the observed data are both necessary.

There are several useful maps:

- exposure time, or the "exposure map";
- quantum efficiency non-uniformity, or the "QE map" (a detector-local map);
- vignetting map;
- effective exposure map (exposure time weighted by QE);
- background map;

All could be defined as a spatially two-dimensional image in world (e.g., sky or XRCF focal-plane) coordinates. (Since the optics and detectors have energy-dependent efficiency, the energy becomes another necessary dimension on the maps.) Each world pixel in the map is a time-integrated or averaged value of a property of the instrument as "seen" by the world pixel. The exposure time map records the total time that a world pixel was exposed by any detector pixel. The QE map records the average normalization of the detector quantum efficiency each world pixel "saw". The effective exposure map is a QE-weighted exposure time. The vignetting map records the average telescope sensitivity normalization factor "seen" by each world pixel.

These maps are needed for imaging as well as grating observations. With gratings in use, however, it is not the sky coordinates that are the relevant world, but the grating diffraction coordinates.

2 Exposure Map

First consider the exposure map for imaging observations: we wish to know how long each piece of the sky was observed so that we can know, for example, that if one region has twice the counts as another, that something in the sky was shining twice as brightly. If there is a gap or edge in the detector, a region could appear faint because half of the time it's light fell into a detector gap or off the detector.

We define a 2D sky array bigger than the detector by the amount of aspect motion and fill it with zeroes. For each aspect history bin, we transform an outline of the detector onto the sky. Then, we add the frame time to all exposure map pixels which lie inside the projected detector boundaries. For the next frame, the outline will be slightly displaced and a few new sky pixels will be incremented by the frame time, a few will be left alone, and most will again be inside the boundary. When we are done with this process, each sky map pixel will have a value equal to the length of time *any* detector pixel observed it. The bulk of the map will have a value equal to the livetime of the observation. The non-zero values will have the shape of the detector, but with fuzzy edges which drop to zero exposure time.

3 Grating Maps

For the gratings, maps are constructed similarly, but in a different coordinate system. The coordinate system needed is the "Grating Diffraction Plane Pixel Coordinates" (GPC) (or the "Grating Diffraction Coordinates" (GDC); see "ASC Coordinates: Rev 3.0 Draft", Jonathan McDowell, April 16, 1996; or the most recent version at `http://hea-www.harvard.edu/asclocal/sds/CDR2/coords.ps`). This system requires a zero-order location vs time (which is analogous to the source sky position vs time in the imaging case). For HETG, there are then two coordinate systems per source, one for each whisker-pair (HEG and MEG) of an HETG spectrum. Bad URL!

4 Mathematical Description

4.1 Definitions

Basic Coordinates

λ: Photon wavelength [Å], $= hc/E \approx 12.398/E$.

E: Photon energy [keV]; $= hc/\lambda \approx 12.398/\lambda$.

(x_d, y_d): Detector pixels [mm].

(x_w, y_w): World coordinate pixels [mm|rad|keV|Å].

t_i: Time of the observation [sec], for the i^{th} aspect history bin.

Mirror Terms

n: HRMA shell [1]; $n = 1, 3, 4, 6$.

A_n: Mirror geometric area [cm^2].

$H(\theta, E, n)$: Mirror efficiency [1];

$H_0(E, n)$: On-axis mirror efficiency [1]. $H_0(E, n) = H(0, E, n)$.

$V(\theta, E, n)$ Mirror Vignetting function [1]; $V = H/H_0$. Mirror vignetting is expected to be only weakly dependent on energy.

Grating Terms

m: Diffraction order; [1].

$G_n(E, m)$: Grating efficiency [1].

r: Dispersion coordinate [mm]; the distance from zero-order centroid parallel to the dispersion direction.

d: Cross-dispersion coordinate [mm]; the distance of a photon from the mean dispersion line perpendicular to the dispersion line.

Detector Terms

$q(E, x_d, y_d)$ Detector quantum efficiency vs Energy and detector pixel [1].

$\overline{q}(E)$: Average detector quantum efficiency [1].
$\overline{q} = \left\{ \int\int q(E, x_d, y_d)\, dx_d\, dy_d \right\} / \left\{ \int\int dx_d\, dy_d \right\}$

$U(E, x_d, y_d)$: Detector QE uniformity [1]. $U = q/\overline{q}$. If the form of q(E) doesn't depend upon position – except for bad pixels – then U is independent of E. Otherwise, U should be only weakly dependent on energy.

Source and Exposure Terms

t_L**:** Live-Time of the observation [sec].

$S(x_w, y_w)$**:** Signal in world coordinates (such as in sky pixels, or in spectrum wavelength pixels) [cts]. This is integrated over PSF (or is the impulse response as if there were no spatial or PHA blur).

$f(E)$**:** Incident flux [photons $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{keV}^{-1}$]. This is the quantity to be determined.

Aspect Terms

i**:** Aspect history bin [1]

θ_i**:** Source Off-axis angle [rad] during the i^{th} aspect history bin.

For imaging observations, θ_i is the off-axis angle of each detector pixel. For grating observations, θ is the off-axis angle of the zero-order centroid.

Δt_i**:** Integration time [sec], during the i^{th} aspect interval as defined by the aspect solution.

$\vec{\alpha}_i$: The aspect history. In flight, this refers to the pointing of the telescope relative to the sky. At XRCF, it refers to the detector position vs time. For example, in flight $\vec{\alpha}_i$ is a vector comprised of the 3-axis pointing position of the HRMA axis at time t_i (such as roll, pitch, and yaw, or right-ascension, declination, and roll). $\vec{\alpha}$ specifies the position and orientation of the optical axis in world coordinates and allows conversion to local coordinates (such as detector pixel).

4.2 A Note on Dither and Coordinates: XRCF vs. Flight

In flight, dither is the motion of the observatory optical axis relative to the sky. At XRCF, dither is the motion of the detector, relative to the optical axis. In the latter, the source position is constant relative to the optical axis.

The detailed coordinate tranformations from detector pixel, detector position, HRMA axis, and other parameters are dealt with in detail elsewhere (McDowell, ASC Memo; `http://hea-www.harvard.edu/asclocal/sds/CDR2/coords.ps`). Here we will refer to the tranformations heuristically, and leave the details to the coordinate transformation library.

The world pixel coordinate is referred to as (x_w, y_w), or for short, $\vec{p}_w$. And similarly for the detector pixel, $\vec{p}_d = (x_d, y_d)$. The world coordinates of any detector pixel depend on the pixel coordinates and the transformation. The dependence on the world

x-coordinate on other parameters, for instance, will be denoted as, $x_w(x_d, y_d, \vec{\alpha}_i)$, where $\vec{\alpha}_i$ represents a state vector of the transformation for the i^{th} time interval, whether it is the flight aspect solution or the six-axes specification of the "five"-axis-mount. The dependence will be condensed into the notation, $\vec{p}_w(\vec{p}_d, \vec{\alpha}_i)$, which is to be understood as "the transformation from $\vec{p}_d$ to $\vec{p}_w$ given the state $\vec{\alpha}_i$." It should also be understood that $\vec{p}_w$ is (r, d) in grating contexts.

4.3 Application

Given these definitions, we can write expressions for the total counts expected at a given world coordinate in terms of the other quantities. We are ignoring blur or other redistributions, so the counts are what would be detected in a given world bin for a point-source with an impulse PSF (in a general sense — both spatial and PHA).

In the case of imaging observations, *and for a specific interval of constant aspect, i*:

$$\begin{aligned} S_i(x_w, y_w) &= f(E)\Delta t_i \\ &\times A_n H_0(E,n)\, V(\theta_i(x_d, y_d, \vec{\alpha}_i), E, n) \\ &\times \overline{q}(E)\, U(E, x_w(x_d, y_d, \vec{\alpha}_i), y_w(x_d, y_d, \vec{\alpha}_i)) \end{aligned} \tag{1}$$

The dependence of world coordinate upon the detector coordinate and aspect solution is shown explicitly, as is the dependence of θ on the detector coordinate. This is also an expression for a single HRMA shell. For the nest, a suitable sum over n of A, H, and V is required (for XRCF, individual shells or quadrants of shells may be specified).

Ugh! need azimuthal integral on HRC-S for shadowed filters.

For grating observations:

$$\begin{aligned} S_i(r) &= f(E)\Delta t_i \\ &\times A_n H_0(E,n)\, V(\theta_i(x_d(r=0,d=0), y_d(r=0,d=0), \vec{\alpha}_i), E, n) \\ &\times G(E,m) \\ &\times \overline{q}(E)\, U(E, r(x_d, y_d, \vec{\alpha}_i), d(x_d, y_d, \vec{\alpha}_i)) \\ &\times \delta(E - E(r))\delta(d). \end{aligned} \tag{2}$$

Here, we include the grating efficiency, G, and transform the detector pixels into diffraction coordinates, (r, d), again for the specific aspect interval. But here, the off-axis angle, θ, is for the zero-order centroid, $(r, d) = (0, 0)$. The final factors, δ, are delta functions, which select the particular energy at the diffraction coordinate, r, and the centerline of the diffraction, $d = 0$ (the spread in d is due to the astigmatic PSF, and here we are interested in counts at a particular r, regardless of how they are spread). Here, we again need an appropriate sum over mirror shells.

Those terms which change in time due to aspect are:

- U is largely uniform, but can switch between two states, such

For HETGS or LETGS, there are only two possible discrete states for any pixel, such as between two CCD quadrants, two different CCDs, or between two HRC-S filters. For ACIS-I, there could be up to four different QE functions seen by one world pixel.

as when dither crosses quadrant or chip boundaries of ACIS or filter boundaries of HRC-S, or at bad pixels. If the detector QE *shape* doesn't depend on E, then U will be independent of E.

- V in principle varies with source position in the field, but it is probably so slowly varying that it can be considered constant over the scale of dither. For imaging observations, V then becomes independent of the detector pixel, and only depends on the world pixel (and energy). For grating observations, V only depends on the zero-order's off-axis angle (and energy). Explicit functional dependence is maintained in anticipation of slew processing, in which a source moves across a large field and vignetting changes appreciably with time.

4.4 Maps: Summation over Time

To define useful map data products, we can consider summations over the time-index (still ignoring the PSF). In the imaging case:

$$\begin{aligned} S(x_w, y_w) &= \sum_i S_i(x_w, y_w) \qquad (3) \\ &= f(E) A_n H_0(E, n) \overline{q}(E) \\ &\times \sum_i V(\theta_i, E, n) U(E, x_w(x_d, y_d, \vec{\alpha}_i), y_w(x_d, y_d, \vec{\alpha}_i)) \Delta t_i. \end{aligned}$$

We identify the second summation as the *effective exposure map*, $T(E, x_w, y_w)$. (If V is constant over the scale of the dither during the observation, then it can be removed from the sum.) This term, T, is analogous to the *flat-field* in optical imaging.

$\theta_i = \theta(x_d, y_d, \vec{\alpha}_i)$

For XRCF shuttered-HRMA observations, $A_n \rightarrow A_{nq}$, where q indicates that the geometric area is for the quadrant, only.

For grating observations:

$$\begin{aligned} S(r) &= \sum_i S_i(r) \qquad (4) \\ &= f(E) A_n H_0(E, n) G(E, m) \overline{q}(E) \\ &\times \sum_i \{V(\theta_{i0}, E, n) U(E, r(x_d, y_d, \vec{\alpha}_i), d(x_d, y_d, \vec{\alpha}_i)) \Delta t_i\} \\ &\times \delta(E - E(r)) \delta(d). \end{aligned}$$

Again, the second summation term can be identified as an effective exposure map. Application of the δ-functions converts it into a one-dimensional object, the effective exposure time at each r-dispersion coordinate (which is proportional to wavelength). The two-dimensional form $(T(E, r, d)\delta(E - E(r)))$ is necessary for determining the relative sensitivity in the two-dimensional image.

$\theta_{i0} = \theta(x_d(r = 0, d = 0), y_d(r = 0, d = 0), \vec{\alpha}_i)$

4.5 Simplified Cases

Imagine for the moment that we have processed a set of data and constructed the effective exposure map defined by T. Then there will be large regions of the map where

- Δt is uniform and equal to the live-time, t_L;
- U is uniform and equals 1.

Thus, for those large regions, we can say:

$$S(x_w, y_w) = f(E)\, AH_0(E)\, \overline{q}(E)\, T(E, x_w, y_w) \tag{5}$$

and we can trivially determine f. The factors multiplying f may be though of as an *effective area map*.

Forgive my units! S is implicitly dependent on energy, or there should be an integration over energy on the right-hand-side.

5 Specific Configurations

Different instrumental configurations allow different approximations and simplifications to the maps. For each major configuration, an explicit version of the appropriate map will be derived.

5.1 Summary of Configurations

1. XRCF
   (a) only one X-ray source in the field
   (b) V is independent of time, and is defined by HRMA polar and azimuthal angles, and detector position.
   (c) HRMA shutters modify range of n, A_n
   (d) HXDS
      i. No dither.
         A. FPC/SSD — single pixel detectors.
         B. HSI — imaging detector.
   (e) Science Instruments
      i. FAM dithers detector position.
2. Flight
   (a) possibility of multiple sources in field.
   (b) No shutters - all quadrants open, only 2 shell-pairs for HETG.
   (c) Pointed Mode
      i. Spacecraft dithers pointing direction, and thereby source off-axis angle(s).
      ii. V is effectively constant over dither scale
   (d) Slew Mode
      i. Pointing undergoes large-scale change.
      ii. V has large change over observation.

5.2 XRCF: SSD/FPC

The XRCF/HXDS Solid-State Detector and Flow-Proportional Counter (SSD and FPC) are single-pixel detectors. Therefore,

$$U = 1$$

and so

$$\begin{aligned} T(E, x_w, y_w) &= V(\theta, E, n) \sum_i \Delta t_i \\ &= V(\theta, E, n) t_L. \end{aligned} \tag{6}$$

Here, (x_w, y_w) specifies a single position (which is (r, d) in the grating case). For the grating case, θ is for the zero-order, not the equivalent world coordinate of the detector's position.

5.3 XRCF: HSI

Since there is no dither of the HSI at XRCF, the off-axis angle is not a function of time, so

$$\theta_i(x_w, y_w) \to \theta(x_d, y_d, \vec{\alpha}).$$

The effective exposure map becomes

$$T(E, x_w, y_w) = V(E, \theta(\vec{p}_w(\vec{p}_d, \vec{\alpha})))\, U(E, \vec{p}_w(\vec{p}_d, \vec{\alpha})) t_L.$$

For the grating case, the relevant world coordinate is (r, d), and θ is for the zero-order position.

5.4 XRCF: Science Instruments

The XRCF dither is detector motion. This means that θ – the off-axis angle of the source – does not depend on t_i, but only depends on (x_w, y_w) and is constant at any given (x_w, y_w). It is the transformation of (x_d, y_d) to (x_w, y_w) which depends on time through the detector dither history. In this case, we re-define $\vec{\alpha}_i$ to be the detector position at time interval i, instead of the HRMA axis vector relative to the source. (see Section 4.2)

Imaging Mode

$$T(E, x_w, y_w) = V(\theta(x_w, y_w), E, n) \sum_i U(E, \vec{p}_w(\vec{p}_d, \vec{\alpha}_i)) \Delta t_i.$$

Dispersed Mode

$$T(r, d) = V(\theta_0, E, n) \sum_i \{U(E, r(\vec{p}_d, \vec{\alpha}_i), d(\vec{p}_d, \vec{\alpha}_i)) \Delta t_i\}\, \delta(E - E(r)).$$

5.5 XRCF: HSI or ACIS-2C Mosaics

Some grating HSI observations are taken with the HSI stepped to several discrete locations in order to sample larger portions of the spectrum than are available in the detector field. These can be handled in the same way as outlined in the previous paragraphs on the science instrument imaging and dispersed modes. The only difference is that there will be fewer positions (and elements in the summation) and less overlap in detector area between steps.

5.6 Flight

5.6.1 Dither Mode

One simplification that can be made during normal observations is that the vignetting does not depend on time. This is true if the scale of the dither does not appreciably change the off-axis angle, so that θ is only a function of world coordinate and not time. In this case, we can take V out of the summations in equations 3 and 4.

5.6.2 Slew Mode

During a slew, the off-axis angle of any source changes substantially during the observation. In this case, the full functional dependence as described in Section 4.4 is needed.

A Grating Algorithm Outline

To make the grating effective exposure map, $T(r,d)$:

1. Create destination array, $T_0(r,d) = 0$.

   (a) Define grid for dispersed portion of a spectrum (HEG, MEG, or LEG, plus or minus side). Should be large enough to accomodate the extraction mask - roughly $r =$ 0–80 mm for HETG, or 0–160 mm for LETG; $d =$ -0.2—0.2 for HETG, -0.4–0.4 for LETG. Pixel sizes should be a few times smaller than the spatial resolution ($24 = \mu$m / few; but a parameter).

2. For each aspect bin, i (or bins of constant aspect)

   (a) read $m = 0$ position

   (b) select a crude window (detector coords ok, ≈ 100 pixels high by chip width — enough to include the spectrum)

   (c) make a list of pixel centers (x_d, y_d) in the region

   (d) compute (r,d) list for each pixel center in list of (x_d, y_d), using zero-order location and aspect history

   Grating Level 1.5 processing computes (r,d).

   (e) compute $E(r)$ list for each r.

   Grating Level 1.5 processing computes $E(r)$.

   (f) interpolate $U(E, x_d, y_d)$ to $E(r)$ for physical pixel (x_d, y_d)

   U is a high-level sub-assembly product.

   (g) [optionally do same for $V(\theta_i, E)$]

   (h) form 2D histogram of (r,d) coordinate list (i.e., an image); keep reverse indices. [need to handle oversampling properly here perhaps form on same grid as (x_d, y_d), then re-sample].

   (i) "anti-alias" image (TBR) set all non-zero pixels to 1 (or determine which are > 1; all should be 1, except where re-gridding causes two old centers to fall into same new pixel)

   Possible numerical issues: anti-aliasing: (r,d) grid may need to be 3 to 5$\times$ finer than (x_d, y_d). a) set only center (r,d) bin? b) set all overlapping (r,d) pixels? c) or 3×3 central overlapping pixels? (probably c; look for Hook ADASS paper for reference)

   (j) multiply each (r,d) pixel by its corresponding U_i (using the reverse indices) [and by V_i, if necessary]

   (k) multiply entire (r,d) image by Δt_i

   (l) add to accumulator $T_{i-1}(r,d)$ to produce $T_i(r,d)$.

Make Effective area map:

- extract $d = 0(\pm\Delta d)$ from $T(r,d)$

  Δd is for numerical range of pixel size about $d = 0$, since it is unlikely to have pixels with $d = 0$, exactly.

- multiply by $A\,H_0\,G\,\overline{q}$ (interpolated to $E(r)$ grid)

B Implementation: Image vs List

While the maps could be implemented as 2D images, this may not be desireable given large fields, large event lists, and mostly smooth images. It may be more efficient to work with lists of local effects and decrement the nominal effective area when analyzing particular sources. Around the boundaries and bad pixels, the individual maps will have to be constructed, but the narrowness of the grating masks makes these very small regions.

The uniformity of U is still to be determined by sub-assembly calibrations. If it is not at all uniform from cell to cell, then the exposure maps may not be so sparse in form after all.

Using lists instead of maps is an accounting problem. The aim is to identify the relatively few detector pixels in detector or world pixels affected by the non-uniformities translated across the world pixels by dither, instead of forming large images with largely redundant information. We concentrate on saving information about the boundaries and in extracting source information, propagate that information into the derived quantities *only if necessary.* If maps are desired, there can be tools to generate them as images, but it is not otherwise necessary to use implement the images in the standard processing.

More TBS ...

C Document History

- Original version: 21 Nov 96 d.p. huenemoerder

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% CfA Directory: /dev/null
% File: Exp_maps.tex
% Author: D. Huenemoerder
% Original version: 21 Nov 96, based on A. Dobrzycki notes of 7/96.
%=============================================
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