6.1. MOUNTING FIXTURE DESIGN

The bow figure is so dominant that, once subtracted, the glass substrates performed at < 1' HPD. An ideal mounting technique should be able to position the mirrors to accuracies of ~ 5 μm regardless of the thickness variations of the pieces, and it should also be able to reduce the bows by slightly bending the piece in the axial direction. The latter characteristics are particularly suitable for the HEFT thermally formed glass.

We have adopted a mounting method that uses axial spacers instead of radial spokes to position the mirrors within a module (43; 44). From a single cylindrical base mount spindle, axial spacer rods are positioned and epoxied axially below each mirror shell segment (Fig. 6.1). The mirrors can be stacked up and epoxied on top of the spacers, and spacers can be stacked up and epoxied on top the mirrors, forming one segment section (like quadrants or sextants) at a time. All the shells are separated and positioned by the spacers. Since all mirror shells are mounted on the same base spindle, including the rear shells, alignment errors are minimized. A tooling ball at each end of the central spindle provides the interface point to the optical bench. This method has important advantages, and in principle, should improve the telescope HPD. The resulting structure is intrinsically strong, and the axial spacers guarantee the dominant bow figure error is controlled by the mounting method. The roundness can be controlled by adjusting the number of spacers per azimuthal angle φ. Since the mirror reflecting surface is in direct contact with the machined profile of the spacers, the shell thickness does not affect positioning accuracy. Some disadvantages are that mirror alignment is not possible after assembly is completed, and individual shells cannot be replaced. The module assembly is described in the next section.
Figure 6.1: Optics module schematic. 3D rendering of the quintant segment optics module design. Some front shells are removed for visualization (44).

6.2 **Prototype Optics Module Fabrication**

I describe the assembly of the optics module after the component shell mirror segments have been formed, cut to size, and coated with multilayers.

The module fabrication process is shown in Fig. 6.2. At the initial stage of fabrication, a set of graphite rods is epoxied on the rigid base mount spindle. Time is allowed for the epoxy to cure, and then the graphite rods are machined to the desired conical shape with a ±2.5 μm precision. Epoxy is applied to the spacers and the first mirror shell is positioned on face down on the spacers. Pressure is applied on the
contact lines to reduce the bows in the mirror shell, and more time is allowed for the epoxy to cure. The next set of graphite spacers is epoxied on top of the first shell. After the epoxy cures, those graphite spacers are machined into the desired conical shape. The machining position is always referenced directly to the tooling ball of the central spindle, so machining errors do not stack up because at the time of machining, the epoxy has fully cured and the underlying structure is rigid. Mechanical metrology is used to monitor assembly. After the second set of spacers is machined, the next mirror shell is positioned and epoxied, and the process continues until 72 shells have been stacked.

Several technical issues were addressed before a prototype was built. The axial spacer technique was developed by starting with flat stacks of glass and by testing the adhesion of the epoxy onto the multilayer-coated glass. The multilayer/glass and multilayer/epoxy adhesion was sufficiently large as to prevent peeling. The epoxy cures at room temperature in \(\sim 3\) hr, and it sets the speed of the process. The graphite spacers can be machined while applying a pressure on the stack which does not induce significant strain. Graphite matches the \(CTE\) of glass well, to \(1 \times 10^{-6}\) K\(^{-1}\), depending on the glass and graphite type. \(CTE\) matching minimizes flexure and internal stresses in the optics module produced by thermal gradients or temperature changes. The mirror stack must be kept clean during machining to avoid reflectivity degradation.

The first optics module prototype shown in Fig. 6.3 was constructed from a cylindrical Al base, and it contained a one-reflection set of five equally spaced uncoated glass shells in a cylindrical geometry. A second prototype shown in Fig. 6.4, consisting of \(20 \times 2\) shells set up for a two-reflection system in a conical geometry, was also produced around a sector base including a tooling ball for alignment (Fig. 6.4).
A full figure and reflectivity characterization was chosen for a third single-reflection prototype which included 5 of the 10 HEFT baseline designs (Table 2.3) for the W/Si graded spacing multilayers on 5 different mirror shells, with radii 8.3, 8.4, 8.5, 8.6 and 8.7 cm, and 20 cm axial length. The Schott AF45 formed glass mirror shells were 300 µm thick. X-ray reflectance, scattering and imaging tests were performed on the third prototype module and are described in the remainder of this chapter.

Figure 6.2: Optics Module Fabrication Procedure (cross section view). Axial graphite rod spacers are epoxied onto the Al sector base. After curing, the spacers are machined with reference to the base. The mirror shell is pressed at the contact points and epoxied into place on top of the spacers. After curing, a new set of spacers is epoxied on top of the glass shell, and the procedure is repeated until the full module is assembled.
Figure 6.3: First *HEFT* prototype optics module. On a single Al sector base, several uncoated mirror shell quadrants were assembled to form a one reflection, single grazing angle system (Photo: W. Craig).

6.3 Testing with a Cu K X-ray Source

6.3.1 Imaging

We describe the measurements of specular and non-specular reflectance at 8 keV, performed at the *DSRI*, with the setup described in Appendices A.1 and A.2. A rotating anode source produces 8 keV X-rays, which reflect on a Si crystal monochromator and transmit through a set of slits to produce a low divergence beam. A $2 \times 0.3$ mm slit was used, with the large dimension oriented along the $\phi$ direction (along the arc-length of the shells), providing a $35''$ FWHM angular resolution. The beam re-
Figure 6.4: Second HEFT prototype optics module. The anodized Al sector base had 20×2 shell quadrants assembled to form a two reflection, double grazing angle system (Photo: W. Craig).

The X-ray beam is reflected on a mirror shell in the prototype module and is detected by a 1-D position sensitive proportional counter, positioned 2.3 m from the module. The prototype module is mounted on a rotating base which allows for φ, α, and translation degrees of freedom. The alignment consists on setting the beam parallel to the outer mirror shell and centering the beam on the outer shell position. This is done by minimizing the beam shadowing vs. α and finding the position where half the beam is shadowed by adjusting the translation stages. The alignment can be verified to a few arcsec by imaging the transmitted X-ray beam through two centered holes along the Al sector.
Table 6.1: Imaging HPD for the shells in the prototype module (in arcseconds) (44). Each HPD was synthesized from a series of pencil beam imaging measurements at various \( \phi \) positions, with full illumination along the length of the mirrors (200 mm). All numbers are for a single reflection optic. Measurements at \( E_x > 8 \) keV are described in Section 6.4.

<table>
<thead>
<tr>
<th>Shell number</th>
<th>( E_x = 8 ) keV</th>
<th>( E_x = 28 ) keV</th>
<th>( E_x = 68 ) keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>31</td>
<td>29</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>36</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>31</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>34</td>
<td>-</td>
</tr>
</tbody>
</table>

base axis. The module is then rotated to the desired angle of incidence. For each measurement, the beam fully illuminates the mirror shell and reflects at \( \alpha \sim 0.1^\circ \). The angular distribution of the reflected intensity was measured by the 1-D PSD and normalized by the direct beam intensity. The HPD of the measured angular distribution was obtained by deconvolving it with the direct beam intensity distribution (the PSF). The results of the measurements performed at various \( \phi \) to sample 50% of the shell surface (44) are shown in Table 6.1.

6.3.2 Scattering

The scattering measurements were performed with a triple-axis diffractometer (Appendix A.2) using a perfect Si(220) channel-cut monochromator and analyzer crystals in a non-dispersive geometry (30). This allows for detection of true scattering for
roughness as small as a few Å on length scales up to 2 mm as well as detection of
figure errors at the arc-second level over the illuminated footprint. With a footprint
of 25 mm, the encircled power diameter of the reflected beam was measured (Fig.
6.5), and a $HPD = 10''$ was obtained from the prototype shells.

From past scattering measurements of the raw, flat glass substrates, it was de-
termined that the glass surface features at spatial wavelengths $< 25$ mm are due to
the manufacturing procedure of the glass (see Section 5.11.2). Thus, the ultimate
imaging capability of the unpolished glass is $\sim 10''$. These results were later verified
at energies $E_x > 8$ keV.

![Figure 6.5: Normalized encircled power diameter at $E_x = 8$ keV for a thermally
formed glass shell in the prototype module. The illuminated area measures
25 $\times$ 2 mm, along the axial direction. The $HPD = 7.6''$, the incidence angle
is 0.4°, and the 90% power diameter is 33'' (44).]
6.4 Testing with Synchrotron X-rays

Imaging, scattering, and reflectivity measurements of the HEFT optics prototype module at the ESRF were performed using the X-ray optics beamline (BM5 - see the setup in Appendix A.3) and the High Energy (ID15A) beamline. At BM5, a detuned double reflection Si(111) monochromator in reflection geometry selected $E_x = 18, 28$ and $34$ keV. A double reflection Si(333), together with an Al absorber to eliminate the 111-reflection, was used to select $E_x = 54$ and $68$ keV. At ID15A, a double reflection Si(311) monochromator in Laue geometry selected $E_x = 65, 80, 90, 100, 115, 150, 158, 160, \text{and } 170$ keV. The spectral monochromaticity was typically $\Delta E_x/E_x = 10^{-4}$. A beam divergence of $14''$ was obtained by adjusting guard slits and was verified from the rocking curve width of the monochromator reflection. ESRF has one of the highest intensity continuous-pulse X-ray beams in the world, allowing for data acquisition in approx. 10 to 50 minutes. The beam intensity, which decayed by $\sim 1$ part in $10^{-3}$ during each measurement, was monitored by a pin diode in ID 15A and by the ring current in BM5. The third optics module prototype was mounted on a ring which provided azimuthal $\phi$ rotation (44; 243; 33; 31).

6.4.1 Imaging

The 2-D imaging instrument was an optical CCD coupled with an X-ray scintillator array, setup at BM5. The camera, located 2.6 m from the optics module, provided an angular resolution of $35''$ FWHM. A $2.00 \times 0.50$ mm X-ray pencil beam was setup to illuminate the full length of the optics shell. Using a $E_x = 28$ keV beam, the spot image for each shell was measured at various $\phi$ positions, and the histogram of HPDs in Fig. 6.7 was obtained. The spot images shown in Fig. 6.6 had no observable
energy dependence from 28 to 68 keV (Table 6.1 also contains data at other energies). Since scattering measurements with 25 mm footprint indicate a $HPD = 8''$ at 28–68 keV energies, confirming the 8 keV data (Section 6.3.2), the observed imaging $HPD$ is completely dominated by large scale figure errors. The observed $HPD$ equals approximately the figure $HPD$ measured by the laser scanner on individual mirrors (Section 3.6), with a bow figure error subtracted (Fig. 6.7). Also, the $HPD$ improved at shell positions $\phi$ near the axial spacers, and it worsened far from the spacers. Therefore, the imaging measurements indicate that the mounting fixture is substantially reducing the bow errors.

To synthesize a global $HPD$ for the optics module from the pencil beam images, the spot images were added. Alignment errors in the setup shifted the spot position on the detector as the optics module was rotated to change $\phi$. Because roundness errors also shift the spot position as a function of $\phi$, a degeneracy arises. Thus, roundness errors may have been subtracted together with the alignment errors. The resulting single-reflection $HPD = 35 \pm 2''$ may not fully account for roundness errors. Extrapolating to two reflections, a factor of $\sqrt{2}$ must be applied (44) in the case of uncorrelated figure, giving an expected $HPD = 49 \pm 3''$ for a conical Wolter-I configuration (Table 2.5). Data at 8, 28, 68 and 170 keV give nearly identical $HPD$ values.

The imaging measurements validated the assembly technique and indicate that a conical Wolter-I optics module with formed glass substrates is realizable with an $HPD$ of 40'', surpassing the 60'' HEFT goal and meeting the Constellation-X HXT requirement. The prototype’s measured $HPD$ is the lowest obtained to date for a hard X-ray optic, and compares favorably to the $HPD = 110''$ obtained for the hard X-ray telescopes in the Astro-E observatory (Table 4.1). Improvements in the figure
with spatial wavelengths > 25 mm on the individual glass substrates can further reduce the \textit{HPD}.

Figure 6.6: Images of a pencil beam reflected off the shell mirrors of the 3rd prototype module. (a) At \( E_x = 28 \text{ keV} \). (b) At \( E_x = 68 \text{ keV} \). The \textit{FWHM} of both images is 26\(^\circ\).

### 6.4.2 Reflectance

Reflectance measurements on the third optics prototype module were made in a \( \theta-2\theta \) geometry, using PIN diodes for both detector and monitor signals. The angular resolution at \( BM5 \) was \( \sim 4\,\text{"} \), and the incidence angle \( \alpha \) was aligned to \( \sim 40\,\text{"} \). The prototype module reflectance was measured for all 5 shells, which have the first 5 coating designs on Table 2.3. The reflectance at 18 to 170 keV was measured (Fig. 6.8), as well as the reflectance for individual shells as a function of \( \phi \) and the direct beam intensity. Fits of these reflectance curves utilized all multilayer parameters: \( d_{\text{max}}, d_{\text{min}}, c, N, \) and \( \Gamma \), plus a single \( \sigma \) for all the interfaces. The goodness of the fits (Fig. 6.9) confirms that the deposited layers have a thickness dependence with depth.
Figure 6.7: *HPD* histogram of a 28 keV pencil beam reflected on all shells of the 3rd optics module prototype. The measurements were taken with a pencil beam fully illuminating each of the shells at a various $\phi$. The cutoff at $\sim 10–14'$ is due to the angular resolution of the detector. The graze angle is fixed at 0.15° (44).

close to the parameterized power-law of Eq. 1.22. The resulting $\sigma = 4.5 \pm 0.2$ Å, with $\sigma = 4.5$ Å at $\phi \pm 17^\circ$, and $\sigma = 5.0$ Å at $\phi \pm 30^\circ$ providing the best fits. Fig. 6.10 shows the thickness uniformity of the multilayer obtained from the variation of the fitted $d_{\text{min}}$ as a function of $\phi$. The $d_{\text{min}}$ values determine the maximum angle at which reflectance is large due to the first order Bragg plateau (31). This maximum angle is $\sim 6$ mrad in Fig. 6.9. The $d_{\text{max}}$ value changes the low-angle reflectance of the Bragg plateau, next to the critical angle, and has a lesser effect. The *HEFT* mirrors will be divided in quintants, or 72° segments. For this arc length, the data in Fig. 6.10
indicates that a < 6% thickness uniformity is achieved. This has a negligible effect in the energy band designated for each multilayer.

The reflectivity measurements at the HEFT operating energies have validated the mirror substrate, coating, and assembly technologies. The W/Si multilayer models agree within a few percent of the measured reflectance (31), indicating that a reliable effective area calculation can be performed for HEFT based on this data. The effective area calculated with a raytrace code and the multilayer models is shown in Fig. 6.11. The calculated area is a few percent above the original HEFT design.

6.5 SUMMARY

A single reflection prototype with $HPD = 35''$ indicates that the glass and mounting technology can achieve the goals of HEFT and Constellation-X HXT. The prototype shell’s multilayer reflectance, and thus the telescope effective area, meets the expectations from the scattering models to $\sim 5\%$, thus, each hard X-ray optics module will meet the designed telescope sensitivity.

Improvement of the $HPD$ requires both single mirror shells with better figure and a more accurate assembly. Single mirror shells may need annealing before forming, especially since cutting can introduce internal stress near the edges. To achieve a Wolter-I telescope with $HPD < 30''$, airborne dust particles of $\gtrsim 1 \mu m$ diameter are a concern, since they may affect the conformance of the glass to the precisely machined graphite spacers. Other improvements on the mounting technique are possible. A finite element analysis calculation of the bending of each mounted mirror shell is now underway. These calculations will be useful to optimize the number of spacers and their locations. An estimate of the safety factor of the assembly may
also be performed since the ultimate strength of the glass may be reached by pieces weakened by microcracks which form during processing.

Further experimentation with thermal forming techniques will determine if the technology may become viable for future soft X-ray telescopes. The weight and microroughness requirements of these future telescopes are met by the formed glass. The remaining hurdle is meeting the $HPD < 15''$ specification. The best prospects for achieving this involve forming polished flat sheets of glass and a forming technique in which the glass conforms to the mandrel to $< 1 \mu m$ accuracy. The mandrel must be polished to the telescope mirror shape to a similar accuracy, but a low-microroughness mandrel is not required, as it would be for an epoxy replication technique.
Figure 6.8: Reflectivity of shell 3 of the 3rd optics module prototype, offset by one decade for clarity. The reflectivity as a function of grazing angle is shown for various $E_x$ (1 mrad $\simeq 3.4'$), and the operating angle range is indicated. The first "bump" on the figures at small angles corresponds to the critical angle of the material, while oscillations at larger angles are due to coherent reflection in the multilayer. These pencil beam measurements were performed at ESRF (31).
Figure 6.9: Reflectance of five multilayer designs at 34 keV and model, for the 3rd optics module prototype. The reflectance vs. $\alpha$ for designs 1–5 (Table 2.3) shows good agreement with the theoretical fits (31). The reflectivity curves have a relative offset of one decade for clarity.
Figure 6.10: Multilayer uniformity: minimum spacing $d_{\text{min}}$ variation with $\phi$, for a shell in the 3rd optics prototype. A 5% non-uniformity is observed in a $\pm 40^\circ$ region on the optic, as obtained from fitting the reflectance data for each $\phi$ (31).
Figure 6.11: Effective Area $A_{\text{eff}}$ for $HEFT$ validated by the characterization of the prototype optics module with W/Si multilayers. The $A_{\text{eff}}(\theta)$ from raytrace calculations is shown, for sources off-axis by an angle $\theta$ (1 mrad $\simeq 3.4'$). A 3.5 g cm$^{-2}$ air column was included (31).
Part II

A Theoretical Model of X-ray Emission from Accretion Disks
Chapter 7

Photoionized Disks in Low-Mass X-ray Binaries

7.1 Overview

The most efficient radiative mechanism known in the Universe is due to dissipative mass infall onto neutron stars or into black holes. When a large amount of gas falls onto such compact objects, it carries too much angular momentum to fall directly, and an accretion disk is formed. Most of what is known about accretion disks is based on the measurements of X-ray fluxes and their time variability. Gravitationally red-shifted line emission suggests accretion disks are bright signposts around many black holes, so understanding disks is essential for future studies that probe the region near the black hole horizon.

This part of my thesis is a study of accretion disks via high energy-resolution X-ray spectroscopy. X-ray spectra contain valuable information on the accretion disk
structure, dynamics and physics, and they provide a window into the physics of photoionized gas and its phase transitions. The disk plasma, at $10^5$–$10^7$ K, cools through atomic line emission that can be detected with space-borne X-ray observatories such as Chandra and XMM-Newton. Modeling the equilibrium state of the plasma and the radiation transfer within the disk allows a calculation of the X-ray spectrum. The synthetic spectrum can be compared to data, in this case from a stellar-size accretion disk. The model spectrum is unique in that it is calculated purely on physical, and not just phenomenological, grounds.

In this introductory chapter, I describe the binary star systems which are studied, the analytic models developed for accretion disks, the previous work on the disk structure and X-ray spectra, and the relevant atomic processes. In Chapter 8, I describe a new model of the ionization structure of a disk which is heated from above by the radiation from a central compact object, and I obtain its radiative recombination emission spectrum. The disk model is compared in Chapter 9 to recent published data from the EXO 0748-69 binary system, among others, for which a 5–35 Å spectrum was obtained with XMM-Newton and Chandra. A revised picture of an accretion disk atmosphere and corona based on the model and the observed data is discussed, as well as predictions for future observations.

### 7.2 Low-Mass X-ray Binaries

Low-mass X-ray Binaries (Section 7.2) are an ideal system to study X-ray emission from accretion disks (Section 7.4), because of their well known geometry and radiation environment. The most prominent and diagnostically-rich emission mechanism in these objects is radiative recombination (Section 7.3.1), which is modeled along with
7.2. LOW-MASS X-RAY BINARIES

a plasma and disk structure code in the next chapter. The state of neutron star and
black hole accretion disk modeling is reviewed.

An X-ray binary (XRB) consists of a compact object—a neutron star or black
hole primary—that is accreting material from another (secondary) star. In the Galaxy,
about 200 XRBs have been catalogued (220) and grouped into two major subdivi-
sions: low-mass X-ray binaries (LMXB; with a main-sequence star secondary of mass
$M_2 \lesssim 1 M_\odot$) and high-mass X-ray binaries (HMXB; with $M_2 \gtrsim 10 M_\odot$ secondaries),
each with a number of subclassifications in use in the current literature (220; 238).
A main-sequence star is one which is powered by fusing hydrogen into helium.

In the broadest astronomical context, XRBs owe their significance to their sta-
tus as products of exotic stellar evolution. Both neutron stars and black holes can
originate from the gravitational collapse of a massive star, which occurs after the
massive star has exhausted its nuclear fuel. Neutron degeneracy pressure is sufficient
to balance the gravitational pressure for all but the most massive progenitors, which
can collapse further to produce a black hole.

XRBs have been studied for nearly four decades. Early optical spectral analyses
revealed their binary nature (21; 109; 110). The best studied XRBs typically have
high accretion luminosities ($L_x = 10^{36} - 10^{38}$ erg s$^{-1}$), owing to the ready availability
of matter to accrete (large $\dot{M}$) and to the compactness of the region in which kinetic
energy is converted into high-energy radiation ($L_x \propto \dot{M}/R_1$, where $R_1$ is the size of the
primary, and $\dot{M}$ is the mass accretion rate). XRBs are the brightest point sources in
the galaxy in X-rays (Fig. 7.1). Using light travel time arguments, the rapid ($\Delta t \lesssim 1$
s) variability of $L_x$ can be used to constrain the source size to $R_1 \lesssim c\Delta t \sim 10^{10}$ cm.
A series of review articles covering most of the major topics associated with XRBs is
contained in Lewin, van Paradijs, & van den Heuvel (118). A number of earlier review
Figure 7.1: *ROSAT* all-sky survey intensity map in Galactic coordinates, from 0.1–2.4 keV. The brightest (white) sources are X-ray binaries, clustered near the Galactic plane (Max Planck Institute for Extraterrestrial Physics).

Articles concerning the specifics of *XRBs* are in the literature (145; 238; 237; 235).

For mass transfer to occur in *LMXBs*, the binary separation must be sufficiently small to allow the primary’s gravitational field to distort the secondary star. An accretion disk mediates mass transfer, the disk being fed by Roche lobe overflow (Fig. 7.2). An effective equipotential surface can be constructed in the co-rotating center of mass frame, if the sum of the gravitational potential and centrifugal effective energy is constant. The Roche lobe is defined as the effective equipotential surface which intersects the inner Lagrangian point in Fig. 7.2. When gas from the secondary fills its Roche lobe, it moves ballistically to impact and form the accretion disk around the
primary. In accretion disks, the annuli are to a good approximation in Keplerian orbits, with projected velocities of \( \sim 10^3 \sin i (M/M_\odot)^{1/2}R_{10}^{-1/2} \) km s\(^{-1} \), for inclination angle \( i \), where \( R_{10} = R/(10^{10} \) cm).

Figure 7.2: Roche lobe diagram of a binary system. This system consists of a compact object (such as a neutron star, white dwarf, or black hole) which is accreting matter from a main-sequence stellar companion (artist’s rendering).

For \( LMXBs \) with neutron stars such as the system shown in Fig. 7.3, the neutron star masses are always \( M_1 \approx 1.4M_\odot \) within measurement error. Some neutron star primaries pulsate in the X-ray band. Doppler shift analysis from X-ray pulsars established that they are located in binary systems (191; 209).

Even with the highest angular resolution available with X-ray telescopes, \( XRBs \) cannot be spatially resolved, as they subtend angles of roughly 100 \( \mu \)arcsec at a dis-
tance of 1 kpc. The X-ray spectrum measured at Earth is thus a composite of spectra formed over a large volume and, presumably, over a wide range of physical conditions. LMXBs radiate most of their energy in the X-ray band, in particular between \( \sim 1 \)–20 keV, but they also emit radiation throughout the rest of the electromagnetic spectrum.

![Diagram of a low-mass X-ray binary system](image)

Figure 7.3: Schematic of a low-mass X-ray binary (LMXB) system with a neutron star primary. Cross section view. The inclination angle \( i \) is defined.

If one assumes that a featureless continuum is emitted in the immediate neighborhood of the neutron star or black hole, then one can think of the interaction of the continuum radiation with the accreting gas as modifying the continuum. The continuum from the compact object is not expected to contain any strong electronic transitions, since the gas is completely ionized. The magnitude of the continuum deformation varies wildly from source to source, and, for a given source, usually varies on observable time scales as well. The X-ray continuum then interacts with circum-source material, which provides, in principle, a set of handles that can be used to construct a physically and geometrically consistent model of the source.
7.2. LOW-MASS X-RAY BINARIES

I review the spectral emission from LMXBs. First, I mention line emission in
the optical and ultraviolet and detections of likely line emission in the X-ray band.
Second, I review X-ray observations in LMXBs preceding the the launch of the
Chandra and XMM-Newton observatories. These X-ray observations point to the
presence of an extended emitter, such as an accretion disk and a hot corona.

7.2.1 OPTICAL AND UV LINE EMISSION FROM LMXBS

There is evidence that X-rays from the LMXB central source are reprocessed by
the accretion disk into the optical and ultraviolet (219), similar to the situation in
cataclysmic variables (158; 157). Optical and ultraviolet emission of the LMXBs is
quite weak. In the optical, the Balmer lines H\textalpha and H\beta are strongest, followed by
He II $\lambda$4686 and the Bowen C III/N III blend at $\lambda$4640 (196). The sensitive ultraviolet
measurements required to detect line emission and the large reddening of sources have
limited the measurements made in LMXBs. Only bright, unabsorbed sources such
as Sco X-1 and Her X-1 have yielded rich line spectra in the ultraviolet, including
lines from He II, C IV, O VI, O V, N V, and Si IV (17; 100; 16). The emission
lines are broadened by the projected Keplerian disk velocity, although narrow line
emission components are also observed, presumably due to X-ray reprocessing on the
secondary. The X-ray to optical+UV luminosity ratio varies from 10–100, depending
on the source, supporting the reprocessing origin of the optical and UV. Still, the
observed optical and UV fluxes are much larger than the expected luminosity of
main-sequence stars with $M_2 \lesssim 1M_\odot$. Visible light spectra and light curves taken
during several orbits can also provide the stellar masses, orbital separation and period.
7.2.2 X-RAY CONTINUUM IN LMXBs

The X-ray spectra of XRBs are almost completely dominated by continuum radiation. In LMXBs, where the lines are produced near the surface of an accretion disk, the solid angle subtended by the disk at the continuum source is $\Delta \Omega / 4\pi \sim 0.1-0.2$ (Section 8.5). Thus, only a small fraction of the source X-ray luminosity leaves the system in the form of discrete X-ray emission lines.

In LMXBs, single-component continuum models usually do not provide adequate fits to the data (239). The continuum spectrum $F_{\nu}^* \ (\text{in units of photons cm}^{-2} \ \text{s}^{-1} \ \text{keV}^{-1})$ can be typically fit by

$$F_{\nu}^* = A_1 \nu^{-\Gamma} \ e^{-\nu/\nu_c} + A_2 B_{\nu}(T_{bb}),$$

with a power law photon index in the range $\Gamma = 1.2-2.0$, normalization constants $A_i$, an exponential cut-off at energies of $h\nu_c \sim 10-30$ keV, and a blackbody component $B_{\nu}$ with temperature $kT_{bb} \sim 1$ keV (145). Electron cyclotron absorption or emission features at $\sim 20-80$ keV have been observed in a subset of the LMXBs and HMXBs which have X-ray pulsars, implying magnetic fields of $B \sim 10^{12}$ G (215; 230). Since the measured continuum shapes are quite featureless, many emission models are able to fit the data (218). For example, a thermal bremsstrahlung model is indistinguishable from a cut-off power law model.

7.2.3 X-RAY LINE EMISSION FROM LMXBs

With the exception of Fe K emission in the 6.4-7.0 keV range (4), discerning X-ray line emission in LMXBs has been challenging, owing to limitations in sensitivity and spectral resolving power, as well as the difficulties associated with attempts to
7.2. LOW-MASS X-RAY BINARIES

extract line emission from data dominated by intense continuum emission. Still, measurements obtained with the *Einstein* Objective Grating Spectrometer (226), the *ROSAT* Position Sensitive Proportional Counter (192), and the *ASCA CCD* imaging detectors (4) have shown that the spectra of a large fraction of bright *LMXBs* exhibit line emission. X-ray lines at $\sim 1$ keV are often mixed with various species, so that only the brightest *LMXBs* had clear line identifications, as in the case of Ne X Ly\(\alpha\) in 4U1626-67 (3), Fe L in Sco X-1, or O VIII Ly\(\alpha\) & Ly\(\beta\) in 4U1636-53 (226).

The X-ray line emission presumably arises as the result of irradiation of the disk by the X-ray continuum, producing an extended source of reprocessed emission. Evidence of X-ray emission from extended regions in *LMXBs* comes from the spectral variations during ingress and egress phases of eclipses, and during rapid intensity fluctuations known as dips. Most dips, which are observed to precede eclipses, are thought to result from variable obscuration and attenuation of the primary continuum by material near the outer disk edge, which has been thickened due to impact of the accretion stream with the disk (233; 63). Dips that are uncorrelated with orbital phase can be produced by orbiting clouds crossing the line of sight, as shown in Fig. 7.3. A cloud larger than $\sim 10^6$ cm can obscure the X-rays from the neutron star. Hard X-ray emission, presumably originating in an accretion disk corona (*ADC*), and representing a few percent of the non-eclipse flux, remains visible during mid-eclipse in several *LMXBs*, implying that the *ADC* is larger than the secondary star (234; 133). *LMXB* spectra during eclipses or dips may *harden* or *soften*, i.e. the proportion between hard ($\sim 3$--10 keV) and soft ($\sim 1$--3 keV) X-rays changes. Most sources harden during dips (155), consistent with photoelectric absorption, but there are exceptions like the softening of X1624-490 (35), and an unchanging X1755-338 (236; 34). Sources such as X0748-676, X1916-053, and X1254-690 show evidence
for an unabsorbed spectral component (155; 36) during dips, revealing an extended source of X-rays which is larger than the $ADC$. These soft X-rays are likely radiation reprocessed in the accretion disk. Dip ingress/egress times indicate $ADC$ sizes in the $10^9-5 \times 10^{10}$ cm range, a factor of a few smaller than the accretion disk sizes calculated from typical orbital parameters (38). A soft X-ray emission component distinct from the hard X-ray continuum has also been interpreted as absorption edges or line emission in some LMXBs (156; 37). The identification of the accretion disk X-ray emission, which has thus far been inconclusive, requires both higher energy resolution and throughput, plus a quantitative theoretical prediction of the X-ray emission from the disk.

### 7.3 Photoionized Plasmas

In this section, I discuss the basic atomic processes of high-temperature astrophysical plasmas, an outline of the solution of the ionization equilibrium state and thermal balance, and the plasma diagnostics that can be useful in determining the dominant ionization mechanism, the electron temperature, and the gas density. Part of this section has been edited from a recent paper by Liedahl et al. (125), and a more detailed discussion may be found elsewhere (124). I describe the most relevant processes for a plasma that is predominantly heated by an external radiation field, as is the case for LMXBs. Since the observed emission must originate where the photon escape probability is near 1, it is a good approximation to assume a situation where the plasma is nearly optically thin.
7.3. PHOTOIONIZED PLASMAS

7.3.1 RELEVANT PLASMA PARTICLE INTERACTIONS

The observed spectrum in LMXBs is the result of four main types of processes: continuum absorption, discrete emission and recombination continuum emission, resonant scattering, and Compton scattering. The X-ray irradiated gas in the vicinity of the continuum source re-radiates lines and continua characteristic of the local ionization state and temperature. The resulting composite spectrum is further modified by its passage through the interstellar medium. Since most LMXBs lie near the Galactic plane at distances \( \gtrsim 1 \) kpc, this is significant, in effect extinguishing the flux from most sources at wavelengths longward of 20–30 Å. This cutoff corresponds to a typical interstellar hydrogen column density of \( N_H \sim 10^{22} \) cm\(^{-2}\), which commonly varies by one or two orders of magnitude from source to source. The emission-line spectroscopy is emphasized in this chapter, and, for the most part, the discussion is restricted to the 1–35 Å band, since there are no observable atomic features shortward of 1 Å (with the notable exception of Landau transitions in high magnetic fields), and most XRBs cannot be observed in the X-ray band at wavelengths longward of 35 Å. The key emission processes are briefly described below.

To generalize each of the reaction types, core denotes an atomic configuration and by \((\text{core}) \, nl\) the same configuration modified by the addition of an electron described by the quantum numbers \(n\) and \(l\). The symbols \(e\) and \(h\nu\) are used for free electrons and photons, respectively. Any process which results in the emission of a photon will cool the gas, since the optically thin regime applies. Compton scattering and bremsstrahlung, not strictly atomic processes, are listed for completeness. Diagrams for the ionization, recombination, excitation, Compton and bremsstrahlung processes are shown in Figs. 7.4–7.7, where the electronic states of ions are represented in energy-level diagrams (different schematics for the atomic processes can be found in
Sako (184) Figs. 1.20 and 1.21). The dominant photon and particle interactions in a photoionized plasma follow:

![Diagram showing ionization processes](Image)

Figure 7.4: Ionization processes. Schematics similar to Feynman diagrams.

- **Photoexcitation and Resonant Scattering.** Consisting of two steps:

\[(\text{core}) \, nl + h\nu_o \rightarrow (\text{core}) \, n'l' \rightarrow (\text{core}) \, nl + h\nu_o\]

The plasma can intercept continuum radiation at energies corresponding to atomic level separations. A photon can be absorbed, which is the photoexcitation process. Photoexcitation can lead to elastic or inelastic scattering, autoionization, or to a radiative cascade. The above equation shows a photon which is subsequently re-emitted at nearly the same energy, but with a new direction. This two-step process is elastic and is called resonant scattering.
Figure 7.5: Diagrams for recombination processes and charge transfer.

Resonant scattering can either add to or subtract from the photon flux propagating toward the observer, depending on the location of the reprocessing gas with respect to the observer’s line of sight to the continuum source. In the plasmas of interest, with near solar elemental abundances, resonant line transitions become optically thick at a column density of order $N_{Z,i} \sim 10^{17}$ cm$^2$, for a given ion $Z^{+i}$.

- **Photoionization.** A photon with an energy $E > \chi$ above the ionization potential $\chi$ dislodges a bound electron:

\[(\text{core}) \, nl + h\nu \rightarrow (\text{core}) + e.\]

This mechanism adds a free electron and heats the gas. An ensemble of continuum photons that photoionize the intervening gas before reaching the line of
sight undergo photoelectric absorption, sometimes producing observable edges (Section 1.2.1). The photoionization cross section near these absorption edges is $\sigma \propto 1/E^3$. Typically, photoelectric absorption becomes optically thick for column densities of order $N_{Z,i} \sim 10^{20}$ cm$^2$.

- **Fluorescence.** A two-step, ionization and excitation process:

  \[ 1s^2 (nl)^N + h\nu \rightarrow 1s (nl)^N + e \]

  \[ 1s (nl)^N \rightarrow 1s^2 (nl)^{N-1} + h\nu_K \]

  Here the initial state “core” is written as the composite of a closed $n = 1$ shell
plus $N$ additional bound electrons. The first step is a K-shell photoionization. In the second step one of the $N$ electrons above the K shell fills the K shell, emitting a characteristic K photon. If the falling electron originates in $n = 2$ (L shell), then the photon is designated $K\alpha$. The probability that the second step above occurs, rather than auto-ionization (the Auger process)

$$1s\ (nl)^N \rightarrow 1s^2\ (nl)^{N-2} + e'$$

is called the fluorescence yield, which scales as $Z^4$. Owing in part to the rapid scaling with $Z$, fluorescence lines from iron are the most commonly observed. In principle, calculating the fluorescence spectrum is a computationally intensive problem. A full calculation of the $K\alpha$ fluorescent spectrum of a few-times-ionized ion would require a determination of the level population distribution of numerous low-lying energy levels, consistent with the local electron density and radiation field. The effect on fluorescent yields is not known in general (88).

- **Compton Scattering.** An electron-photon scatter:

$$e + h\nu \rightarrow e' + h\nu'.$$
In plasmas in which the abundant elements are stripped of their electrons, Compton scattering can regulate the rate of energy exchange between the gas and the radiation field, and can, depending on the Compton depth and the electron temperature, produce substantial modification to the spectrum. When the kinetic energy of the electron is much smaller than the photon energy \( KE_e \ll E \), the photon wavelength shift is \( \Delta \lambda = h(1 - \cos \theta)/mc = 0.024(1 - \cos \theta) \, \text{Å} \), and the gas is heated. The inverse process occurs when the \( KE_e \gg E \), and the gas is cooled.

- **Electron-Ion Impact Excitation.**

  \[(\text{core}) \, nl + e \rightarrow (\text{core}) \, n'l' + e'\]

While electron temperatures in X-ray photoionized plasmas are too low (see section 7.3.3) to effectively populate high-lying energy levels by electron impact excitation \((\text{EIE})\), this process is often the dominant one in transferring population between closely-spaced levels. Important instances of \( \text{EIE} \) are the collisional depopulation of the energy level responsible for the so-called forbidden line in He-like ions \((1s^2-1s2s \ 3S_1)\), and the populating of low-lying excited states in L-shell and M-shell ions.

- **Radiative Recombination** between an electron and an ion:

  \[\text{core} + e \rightarrow (\text{core}) \, nl + h\nu.\]

The outgoing photons are distributed into a continuum—the radiative recombination continuum \((\text{RRC})\)—above the recombination edge \((E = \chi)\), with a width \( \Delta E \approx kT \), where \( T \) is the electron temperature. In an over-ionized
plasma $\Delta E/E \approx kT/\chi \ll 1$, which means that the recombination continuum feature is narrow, and appears “line-like.” To date, the only identified RRC features have been associated with recombination to the ground level of H-like and He-like ions, but more often recombination leaves an ion in an excited state.

- *Dielectronic Recombination.* In this two-step process, the first step, radiationless capture, excites an electron belonging to the initial-state ion—no photon is emitted.

\[
(\text{core}) + e \rightarrow (\text{core})' \, nl
\]

In X-ray photoionized plasmas, the core is excited by way of an intrashell transition, such as $2s \rightarrow 2p$. The energy of the ion following capture lies above the first ionization limit. The ion may, therefore, auto-ionize. For example,

\[
(\text{core})' \, nl \rightarrow \text{core} + e',
\]

which is, in effect, an elastic scatter when combined with the first step. Alternatively, the ion can emit a photon, leaving the ion with an energy below the first ionization limit (*radiative stabilization*), which completes the dielectronic recombination (DR), for example,

\[
(\text{core})' \, nl \rightarrow (\text{core})' \, n'l' + h\nu_o.
\]

$DR$ is not an important process for K-shell ions, but can be the dominant recombination mechanism for L-shell ions.

- *Bremsstrahlung.* A continuum photon is emitted by the acceleration of an electron in the electric field of an ion:
\[(core) + e \rightarrow (core) + e + h\nu.\]

This process cools the gas, while its inverse process heats the gas by continuum absorption.

- **Radiative Cascade.** A spontaneous process:

\[(core) \, nl \rightarrow (core) \, n' \, l' + h\nu.\]

is also an excitation process. Subsequent to recombination or *EIE* into an excited level, the ion decays in a series of spontaneous radiative transitions, until it reaches the ground level. At each step the ion may have access to several decay channels. The relative probabilities attached to these various channels partly determine the ionic spectrum. At high densities, however, *EIE* can interrupt the cascade, possibly resulting in line ratios that differ noticeably from the low-density limit. For ions with L-shell electrons, cascades following both *RR* and *DR* must be considered. Since the post-*DR* stabilized state can have an excited core, cascades that proceed in the presence of a “frozen” excited core are possible, the spectrum of which is distinct from the *RR* cascade spectrum.

Since collisional excitation of high-lying states is exceptionally rare in X-ray photoionized plasmas, radiative cascades following recombination are of paramount importance, which can be contrasted to the case of collisionally-ionized plasmas, where, in the context of line spectroscopy, recombination processes are generally treated as details.

- **Charge transfer.** An exchange of an electron between neutral hydrogen and an ion:

\[(core) + H \rightarrow (core) \, nl + H^+.\]
In spite of the low abundance of neutral hydrogen, charge transfer can be important since its rate coefficient might be orders of magnitude larger than other recombination processes (22). Because of Coulomb repulsion, charge transfer is more efficient in moderately ionized species, which are detectable in the X-ray band through their fluorescence emission.

7.3.2 Photoionized vs. Collisionally Ionized Plasmas

The competition between collisional-ionization cooling and photoionization heating determines the relationship between the ionization state and the electron temperature. Coronal or collisional-ionization requires the electron temperature to be comparable to the ionization potential of all ions, \( kT \sim \chi \). Photoionization heated plasmas, on the other hand, are in a radiation field where photons can have energies that greatly exceed the electron temperature, and it is the radiative energy per gas particle which drives ionization, so that \( h\nu \gtrsim \chi \gg kT \). Thus, a given charge state exists at much colder temperatures \( T \) in a photoionized gas than in a collisionally ionized gas. For a given disk area, the incident radiative energy is much larger than the energy dissipated in the disk (Eq. 7.8). Neutron star LMXBs are thus sufficiently luminous that photoionization dominates.

7.3.3 The Ionization Parameter

The physical conditions in optically thin X-ray photoionized gas are, for a given ionizing spectrum, conveniently parameterized by the ionization parameter. Although there are a number of quantities dubbed “ionization parameter” in the literature, only two variations are discussed. Most of what follows is discussed in terms of \( \xi \) (210), the
use of which is motivated by consideration of the steady-state equations of ionization balance. Also, the parameter $\Xi$ (112) is useful for gases in hydrostatic equilibrium. In steady state the equations of ionization balance for each ion $Z^{++}$ can be written as

$$\frac{\partial n_{z,i}}{\partial t} = n_{z,i+1} n_e \alpha_{z,i+1} + n_{z,i-1} (\beta_{z,i-1} + n_e C_{z,i-1}) - n_{z,i} (\beta_{z,i} + n_e \alpha_{z,i} + n_e C_{z,i}) = 0,$$  \hspace{1cm} \text{(7.2)}

where $\beta_{z,i}$ is the photoionization rate ($\text{s}^{-1}$) of $Z^{++}$, $C_{z,i}$ is the collisional ionization rate coefficient ($\text{cm}^3 \text{s}^{-1}$) of $Z^{++}$, $\alpha_{z,i+1}$ is the recombination rate coefficient ($\text{cm}^3 \text{s}^{-1}$) of ion $Z^{+(i+1)}$, and so on. The terms with $\alpha_{z,i+1}$ and $\alpha_{z,i}$ account for all two-body recombination processes. The coefficients $C_{z,i}$ and $\alpha_{z,i}$ depend on the electron temperature $T$ for any ion $Z^{++}$. Given the photoionization cross-section $\sigma_{PE,\nu,z,i}$ and ionization threshold energy $\chi_{z,i}$ of the ion $Z^{++}$, the photoionization rate for a point source of ionizing continuum can be written

$$\beta_{z,i} = \frac{L_x}{r^2} \int_{\chi_{z,i}}^{\infty} dE \frac{S_E(E)}{4\pi E} \sigma_{PE,\nu,z,i}(E),$$  \hspace{1cm} \text{(7.3)}

where $S_E$ is the spectral shape function, normalized on a suitable energy interval. Denoting the integral in Eq. 7.3 by $\Phi_{z,i}$, Eq. 7.2 becomes

$$\frac{L_x}{n_e r^2} (\Phi_{z,i} n_{z,i} - \Phi_{z,i-1} n_{z,i-1}) + n_{z,i} (C_{z,i} + \alpha_{z,i}) = \alpha_{z,i+1} n_{z,i+1} + C_{z,i-1} n_{z,i-1}.$$  \hspace{1cm} \text{(7.4)}

Let $\xi = L_x/n_e r^2$ (in erg s$^{-1}$ cm), which is called the ionization parameter. Then, assuming $n_{z,i-1} \ll n_{z,i}$ to simplify the argument,

$$\frac{n_{z,i+1}}{n_{z,i}} \approx \frac{\alpha_{z,i} + C_{z,i}(T)}{\alpha_{z,i+1}(T)} + \xi \Phi_{z,i}.$$  \hspace{1cm} \text{(7.5)}

Eq. 7.5 shows that an X-ray photoionized plasma with temperature $T$ is over-ionized with respect to collisional ionization balance, owing to the presence of the $\xi \Phi_{z,i}$ term (Section 7.3.2). The simultaneous solution of the set of ionization equations (Eq.
7.3. PHOTOIONIZED PLASMAS

7.5) and the energy equation (Eq. 8.11) gives the $T(\xi)$ relation. Photoionization dominates in LMXBs, and $C_{z,i}$ can be neglected.

Another, dimensionless ionization parameter can be constructed with the radiation pressure $P_{rad}$ and the proton gas pressure $P_{gas}$ (112). This ionization parameter, $\Xi$, is defined as $\Xi \equiv P_{rad}/P_{gas}$, where $P_{rad} = F_X/c$, with ionizing flux $F_X$ in erg $s^{-1}$ cm$^{-2}$. Note $\Xi = \xi/4\pi c k T$. The new parameter $\Xi$ is useful when the local pressure can be defined. For an optically thin gas, an isobar has constant $\Xi$. In that case, any phase transitions taking place at constant pressure also take place at constant $\Xi$, as is described in Section 8.3.

### 7.3.4 X-RAY PHOTOIONIZATION CODES

Thorough treatments of photoionization codes and their applications can be found in Davidson & Netzer (47), Ferland et al. (58), and Kallman & McCray (97). In short, photoionization codes are used to determine the effect of a radiation field on a gas of specified chemical composition, and the self-consistent effect that passage through the gas has on the radiation field, including the addition of local sources of radiation. This entails, among other things, the determination of the charge state distribution and temperature as functions of $\xi$. Maxwellian electron distributions are assumed, but local thermodynamic equilibrium (LTE) is not. LTE consists of assuming that the atomic level populations are given by the Boltzmann law, based on a locally defined value of the kinetic temperature, which is equivalent to assuming that the local source function is a blackbody function. More broadly, under the LTE assumption, the equilibrium among kinetic and internal degrees of freedom is reached independently of the details of the interaction mechanisms (90). LTE is rarely applicable, owing to the
insufficiently high electron densities and the highly-diluted non-Planckian radiation fields found in XRBs. Calculations are performed assuming statistical equilibrium or detailed level accounting, whereby charge state fractions and level populations are calculated by explicit inclusion of all relevant rates into and out of each energy level. It is found that highly-ionized ions exist over the approximate range 10−10^4 in \( \xi \) (CGS units). Below that range recombination lines fall at energies below the X-ray band, but inner-shell fluorescence line emission is still possible. Above that range, ions are fully stripped, but hydrogen-like recombination emission is still possible. Recombination spectra are formed primarily in the approximate temperature range 10^5-10^6 K.

The plasma code that is used for this investigation (173) relaxes some of the assumptions of the coronal approximation. The coronal approximation consists in assuming an optically thin gas, with ground state-level populations and a Maxwellian particle velocity distribution. Because of the high densities present in the accretion disk atmosphere, the coronal approximation is questionable. The Raymond (173) code computes the level populations including the n=1–4 levels of H and He\(^+\) and seven levels (ground, 1s2s, 1s2p, and 1s3d singlets and triplets) of He\(^0\). Ions of other elements are assumed to be in their ground states, except for the populations of the metastable 2s2p \(^3\)P, 2s2p\(^2\) \(^4\)P, 2s2p\(^2\) \(^5\)S, and 3s3p \(^3\)P levels of Be\(^-\), B\(^-\), C\(^-\), and Mg-like ions, which are used to compute excitation rates. The code is also extended for high optical depth. Each line connected to the ground state is treated in a two-level approximation. The populations of the upper energy level \( u \) and the lower energy level \( l \) and the line power are computed with

\[
\frac{\partial n_l}{\partial t} = n_u n_e q_{u \rightarrow l} + n_u A_{u \rightarrow l} p_{l \rightarrow u} - n_l n_e q_{l \rightarrow u} = 0
\]  

(7.6)

where the radiative transition probability \( A_{u \rightarrow l} \) is reduced by the escape probability
7.3. PHOTOIONIZED PLASMAS

$p_{i\rightarrow u}$, and $q_{i\rightarrow j}$, $q_{l\rightarrow u}$ are the collisional excitation rate coefficients. The approximate escape probability $p_{i\rightarrow u} \simeq 1/(1+\tau)$ is used, where $\tau$ is the vertical optical depth. Over 1000 spectral lines are included in the thermal and ionization equilibrium calculation, most from highly ionized charge states, as detailed by Raymond (173).

### 7.3.5 Spectral Diagnostics in X-Ray Photoionized Plasmas

Spectral diagnostics are used to quantify the physical conditions of a plasma, by measuring line ratios from a single ion, or from the lines of two or more ions. The physical parameters in an $LMXB$ vary spatially, because of the extended nature of the source, which encompasses a broad range of radiation intensities and density, for example. Any physical parameter thus derived is taken from lines emitted from an ensemble of regions within the $LMXB$, and care must be taken to interpret the lines correctly. The line diagnostics can be used to test the nature of the source within a framework of a model containing the structure and dynamics of the $LMXB$. The inference of a physical parameter from the line ratios is thus not as straightforward as it would be for a uniform plasma with a single ionization parameter and radiation spectrum. For the remainder of this section, the plasma diagnostics are introduced in the context of such a monolithic plasma.

A more detailed treatment of X-ray spectroscopy of photoionized plasmas, with emphasis on atomic kinetics, is given in Liedahl (124). A more general presentation of X-ray spectroscopy, including discussions of the relevant atomic rates, can be found in Mewe (138).
Signatures of Dominance by X-Ray Photoionization

Although X-ray line emission in LMXBs is likely to originate in photoionized gas, one still seeks explicit confirmation through examination of spectra. After all, it is conceivable that LMXBs contain regions of, for example, shock-heated gas. The indicator of photoionization dominance is recombination dominance in the emission line spectra of high charge states. Having discerned that the emission mechanism is consistent with pure recombination, one can then appeal to photoionization codes in order to assign a rough value of $\xi$, from the $f_i(\xi)$ relationship, and $T$, from the $T(\xi)$ relationship, both derived for the appropriate ionizing continuum. Fortunately, there are a few easily recognized spectral signatures of recombination dominance, described below.

(1) Presence of RRC. As discussed in §1.3, an X-ray photoionized plasma is over-ionized, i.e., the electron temperature is much less than the ionization temperature. This condition leaves recombination as the dominant level populating mechanism (see Section 1.2). X-ray lines produced by radiative cascades following recombination are accompanied by (often) narrow RRC. If the dominant heating mechanism is not photoionization, though, as in the hot LMXB corona, which is dominated by Compton heating, broad RRC in H-like ions could be observed (Section 8.4). In the optically thin limit the ratio is only a weak function of temperature. For H-like ions, for example, the ratio $I(1s-2p)/I(RRC)$ is approximately 1.3 (123). Since RRC shapes are determined by $T$, RRC provide a relatively model-independent temperature diagnostic, and can, in principle, be used to check the theoretical $T(\xi)$ relationship. A well-resolved RRC shape can also prove that the line is produced at regions within a range of temperatures.
(2) Large $G$ ratio in He-like ions. The He-like $G$ ratio is defined as the intensity ratio $(x + y + z)/w$ (Fig. 7.8), where $x$ and $y$ are the intercombination lines $1s^2 \, ^1S_0 - 1s2p \, ^3P_{2,1}$, $z$ is the forbidden line $1s^2 \, ^1S_0 - 1s2s \, ^3S_1$, and $w$ is the resonance line $1s^2 \, ^1S_0 - 1s2p \, ^1P_1$ (65; 66). A large $G$ ($\approx 3-4$) implies that recombination dominates, while $G \approx 1$ implies collisional ionization (166; 124; 10; 164). Thus a visual examination of the lines is sufficient to discern whether photoionization or recombination dominate. With velocity fields of a few $\times 1000$ km s$^{-1}$, line broadening can cause $x$ and $y$ to blend with $w$, so that $G$ cannot be trivially specified, and other combinations of the
lines may need to be used (151). The $w, x, y, z$ lines of the He-like ions of N, O, Ne, Mg, and Si can be resolved and detected by the Chandra and XMM-Newton gratings.

(3) Relatively weak or apparent absence of iron L-shell emission in presence of K-shell emission from lighter elements. Decades of solar X-ray observations have familiarized spectroscopists with the fact that bright iron L-shell emission dominates the line spectrum of cosmic-abundance plasmas with temperatures in the $10^6$-$10^7$ K range (162). It was shown by Kallman et al. (99), however, that under conditions of over-ionization, iron L-shell emission is overwhelmed by $1s - np$ and RRC emission from K-shell ions of, for example, O, Ne, and Mg. There are several reasons for this. Perhaps the simplest way to look at it is to think of iron L-shell ions as having unusually large collisional rate coefficients, which help to drive the impressive line fluxes observed in collisionally ionized plasmas, but do not come into play at all in photoionized plasmas. These calculations have been borne out in LMXBs by the observation of EXO 0748-69, which is reviewed in Section 9.2. Recombination spectra also affect iron L-shell ratios (121), but the use of these ratios may have to wait for instruments of higher sensitivity.

The $R$ Ratio in He-like Ions

He-like ions are also density diagnostics (65). The useful ratio is denoted by $R$, and is defined as $R = z/(x + y)$, where $x$, $y$, and $z$ were defined in the previous section. The $z$ line is a slow magnetic dipole transition, so that as $n_e$ increases, collisional depopulation of the $1s2s\, {}^3S_1$ level up to the $1s2p\, {}^3P_{2,1,0}$ levels begins to compete with radiative decay to the ground state. The density at which the rates of these two depopulation mechanisms are equal is called the critical density $n_{\text{crit}}$. The dominant
sinks for the outgoing population flux are the three $1s2p\,^3P$ levels, two of which can decay to ground, producing the $x$ and $y$ lines. Thus the ratio $R$ decreases with increasing $n_e$.

![O VII](image)

Figure 7.9: The density dependence of the $R = z/(x + y)$ line ratio, for the O VII (He-like) ion. The dependence of $R$ on temperature is weak (55).

The density dependence of $R$ is well approximated by $R = R_o [1 + (n_e/n_{\text{crit}})]^{-1}$, where $R_o$ is the zero-density limit of $R$ (Fig. 7.9). The variation of $R$ is greatest for the two orders of magnitude in $n_e$ centered on $n_{\text{crit}}$. Below this range, one would observe $R \approx R_o$, and assign an upper limit to $n_e$. Above this range, $R \approx 0$, and a lower limit can be assigned.

The ground state transitions of He-like ions of C, N, O, Ne, Mg, Si, S, Ar, Ca, and Fe fall into the X-ray band, and they can be detected with Chandra and XMM-Newton. Both $R_o$ and $n_{\text{crit}}$ vary with $Z$ (165), and obtaining their theoretical values
requires detailed solutions of the rate equations (10; 164) In particular, $n_{\text{crit}}$ increases with $Z$, ranging from $\sim 10^9 \text{ cm}^{-3}$ for $\text{C V}$ to $\sim 10^{17} \text{ cm}^{-3}$ for $\text{Fe XXV}$. To the extent that the various He-like spectra are present in a given set of data, density information is available over a broad region of $n_e-\xi$ parameter space.

The prevalence of He-like ions in highly-ionized plasmas, the relative simplicity of calculating He-like spectra, and their sensitivity to plasma conditions has made them the pre-eminent diagnostics in X-ray spectroscopy. However, a complication that was noted early on in the study of He-like spectra is now being revisited, one which may compromise the use of the $R$ diagnostic in some systems, notably $LMXB$s. The collisional transitions that are crucial to the density sensitivity of He-like ions, $1s2s \, {}^3S_1 \rightarrow 1s2p \, {}^3P_{0,1,2}$ are all electric dipole transitions, and can be driven equally well by photons. The wavelengths corresponding to the energy separation of the $1s2s \, {}^3S_1$ and $1s2p \, {}^3P_1$ levels, to take an example, are 1634 Å, 1273 Å, and 868 Å, for O, Ne, and Si, respectively. A bright UV source in the vicinity of the X-ray emission-line regions can thus have the effect of mimicking the signature of high electron density.

The possible influence of UV radiation on $R$ was pointed out by Gabriel & Jordan (65) for solar applications, where it was found that only C V should be affected by photospheric UV emission, and by Blumenthal, Drake, & Tucker (15), who emphasized that, for example, the presence of a moderately hot white dwarf could drive the $R$ ratio to low values. This has obvious implications for $LMXB$s, in which an accretion disk emits copious UV radiation. It was argued by Kahn et al. (101) that the low values of $R$ observed in the isolated O super-giant $\zeta$ Pup are consequences of UV driving, rather than high densities, consistent with expectations that stellar winds are too tenuous for the collisional depopulation of the $1s2s \, {}^3S_1$ level
to matter. However, since the modeled disk density is high enough for the collisional mechanism to work (Section 8.5), and a UV source is also present, a degeneracy occurs. This is the case in \textit{LMXBs}, even if just the local thermal UV disk emission is taken into account (122). The integrated UV luminosity of the disk will take away the ability to discern high densities with He-like ions. What is needed is another X-ray density diagnostic, one that is not susceptible to UV driving, and one for which the critical density lies in an interesting part of parameter space. One such candidate is the Ne-like Fe XVII 17.10/17.05 ratio ($n_{\text{crit}} \sim 10^{13}$ cm$^{-3}$), which appears to be immune to the influence of few-eV Planckian radiation fields (Mauche, Liedahl, & Fournier, in preparation).

### 7.4 Accretion Disk Models

The intense emission from \textit{LMXBs} is attributed to the efficient conversion of gravitational potential energy into radiation, as mass is transferred from an otherwise normal star to a compact object (200). The mass donor (secondary or companion) overflows its Roche lobe (Fig. 7.2), thereby spilling matter through the inner Lagrangian point (\textit{L1}). Having a large angular momentum, the matter accreting through \textit{L1} follows trajectories that cannot directly intersect the compact object. Instead, a disk forms (Fig. 7.10), and a tangential viscosity forces the disk material to spiral in (167).

The viscous mechanism in accretion disks allows accretion to occur by transporting angular momentum outward, matter inwards, and by dissipating gravitational energy into heat inside the disk. Consider a dissipative tangential coupling (i.e. a dashpot) between adjacent ring sections of a disk moving at Keplerian velocities $v_K = \sqrt{GM/r}$. The inner disk, which has higher tangential velocity, will lose angular
momentum and kinetic energy, thus moving into a smaller orbit, while the outer ring will gain angular momentum and kinetic energy, moving into a larger orbit. Thus, angular momentum is transported outwards, allowing material in the disk to accrete. This dissipation mechanism has been identified in theory as a magneto-rotational instability due to the entanglement of magnetic fields from the differential rotation of the gas in the disk, akin to a dynamo effect (7). In resistive MHD models, this mechanism provides the energy dissipation and angular momentum transfer needed to produce mass accretion with self-sustained magnetic fields $B$ that are much smaller than the equipartition level ($B^2 / 8 \pi \ll \rho v^2$), where $\rho$ is the gas density and $v$ is the thermal velocity (7).

Figure 7.10: Schematic of the accretion disk geometry and its velocity field $\vec{v}$. In the standard SS73 model (197), the pressure scale height $H_P \ll r$. 
7.4. ACCRETION DISK MODELS

Shakura & Sunyaev (197), or SS73 hereafter, derived an analytic model of radiatively efficient matter infall in a disk geometry. The viscously dissipated energy is assumed to be locally radiated as blackbody emission (169; 197; 153; 140; 24; 111; 206; 116). The UV continuum of cataclysmic variables is in rough agreement with the above assumption (103; 227; 170). The SS73 parameterization of the viscosity \( \eta \) in terms of the total pressure \( P \), such that \( \eta = \alpha P \), where \( \alpha \) is a constant, provides a zeroth order picture of the disk, but it is likely incorrect in detail. Numerical magneto-hydrodynamic (MHD) models (77) show that \( \alpha \) varies by a few orders of magnitude throughout the disk, but that on average its value ranges from \( \sim 10^{-3} \) to \( 10^{-1} \). Nevertheless, the \( \alpha \)-disk model is one of the simplest and most useful of its kind, especially since many results do not sensitively depend on the value of \( \alpha \). The central disk density, which is the quantity that is most dependent on \( \alpha \), is \( \propto \alpha \). The pressure scale height \( H_P \) of the \( \alpha \)-disk satisfies \( H_P \ll r \), and while the Keplerian velocity is much larger than the local sound speed \( v_\phi = v_K \gg c_s = \sqrt{P/\rho} \), the radial accretion velocity is very subsonic, \( v_r \ll c_s \). The disk is also highly optically thick, \( \tau \gg 1 \), which is required to produce the local blackbody emission. Comparisons of the \( \alpha \)-disk model with observations of XRBs and AGN are presented in Frank, King & Raine (62) and Malkan (127), respectively. Modern accretion flow models, of which the \( \alpha \)-disk is a subset, are described by Chen et al. (27).

The \( \alpha \)-disk model has been accepted with only a few modifications since its inception (1; 26; 126), and a significant fraction of the X-ray continuum in LMXBs is interpreted as thermal emission from the disk. Yet, the presence of non-thermal continuum radiation in all of these sources and line emission from highly-ionized atomic species in many of them is not accommodated by the standard \( \alpha \)-disk model. Moreover, neutron star or coronal thermal emission has made the discernment of disk
continuum emission ambiguous. As such, X-ray line emission is a potentially powerful tool for elucidating the properties of accretion disks.

### 7.4.1 Radiatively Heated Accretion Disks

**Radial Structure**

In LMXBs roughly half of the gravitational potential energy is released in the vicinity of the compact object (i.e., in accretion shocks near the neutron star surface). The flared disk is exposed to this radiation, and will be heated by it. In fact, radiative heating can exceed internal viscous heating in the outer region of the disk. The temperature structure of the disk can thus be controlled by the X-ray field photoionizing the gas, suppressing convection, and increasing the scale height of the disk. Photoionization in the disk is balanced by recombination, which produces line emission.

Assuming that all the viscous heating and radiative heating from illumination by the central source is radiated locally as a blackbody (as in the SS73 model), Vrtilek et al. (225) find, for a geometrically thin disk, and for \( r \gg R_1 \), where \( R_1 \) is the radius of the compact X-ray source,

\[
\sigma T_{\text{phot}}^4 \approx \frac{3GM_1 \dot{M}}{8\pi r^3} + \frac{(1 - \eta)L_\odot \sin \theta(r)}{4\pi r^2},
\]

(7.7)

where \( T_{\text{phot}} \) is the photospheric temperature, \( M_1 \) is the mass of the compact X-ray source, \( G \) is the gravitational constant, \( \sigma \) the Stephan-Boltzmann constant, \( \theta \) is the grazing angle of the incident X-ray flux with respect to the disk surface, and \( \eta \) is the X-ray albedo such that \( 1 - \eta \) is the fraction of X-rays absorbed at the photosphere. The albedo can be derived from optical observations (48). The observable blackbody emission takes place at the photosphere. The first term on the right-hand side of Eq.
7.4. ACCRETION DISK MODELS

7.7 is the energy dissipated within the SS73 disk, and the second term is the radiative heating. The radiative heating term will dominate where

\[ r > 2.3 \times 10^8 \left( \frac{M_1}{M_\odot} \right) \left( \frac{1 - \eta}{0.1} \right)^{-1} \left( \frac{\sin \theta}{0.1} \right)^{-1} \left( \frac{\epsilon_x}{0.1} \right)^{-1} \text{cm}, \quad (7.8) \]

where the X-ray luminosity is written in terms of an X-ray accretion efficiency \( \epsilon_x \), according to \( L_x = \epsilon_x \dot{M} c^2 \). For example, accretion onto a neutron star results in roughly 1/2 of the gravitational potential energy being converted into X-rays, or \( \epsilon_x = G M_1 / 2 c^2 R_1 \). The disk, therefore, is radially divided into an inner region dominated by internal dissipation, and an outer region dominated by external illumination. External radiation will dominate the disk atmosphere energetics for the outer two or three decades in radii, and the local dissipation and magnetic flare heating, if any, can be ignored there.

**VERTICAL STRUCTURE**

The previously discussed disk models rely on averaging physical quantities in the direction perpendicular to the disk plane. However, the radiative recombination spectrum is very sensitive to the radial and vertical ionization structure.

To obtain a high-resolution spectrum of an accretion disk, and in particular one for which the outer layers are X-ray photoionized, several authors have calculated the vertical structure by solving the radiation transfer equations, assuming hydrostatic equilibrium. Models have been applied to AGN and LMXBs in the high-\( L_x \) state, since in the low-state other accretion modes ensue. The radiative transfer is typically simplified by using an on-the-spot approximation and the escape probability formalism. Due to photoelectric absorption and Compton scattering, the resulting ionization structure of the disk is stratified, approximated by a set of zones, each
with a single ionization parameter. The ionization structure of the disk can be solved by using photoionization codes, such as \textit{CLOUDY} and \textit{XSTAR}, to calculate the ionization and thermal equilibrium state of the gas at each zone.

Ko & Kallman (106; 107) calculated the vertical structure of an illuminated accretion disk and obtained the recombination X-ray spectrum for individual rings on the disk. Raymond (173) utilized the calculated temperatures in Vrtilek et al. (225) to calculate a vertical structure and the UV spectrum from the global structure of the disk. Both assumed parameters for \textit{LMXB}s, and gas pressure-dominated disks. Later models of photoionized accretion disks focused primarily on calculating the Fe K\alpha fluorescence emission from \textit{AGN} disks.

Rozanska & Czerny (180; 181) modeled semi-analytically the stratified, photoionized transition region between the corona and the disk. They found that their approximations, which included on-the-spot absorption, matched more accurate radiation transfer codes, including optically thick scattering, for optical depths \( \lesssim 10 \). They also discussed the existence of a two-phase medium, stopping short, however, of calculating an X-ray line spectrum. Nayakshin, Kazanas, & Kallman (146) modeled a radiation-pressure dominated disk and showed that the vertical structure of the disk implied significant differences in the Fe K fluorescence line spectrum compared to that predicted by constant-density disk models (176; 131; 252). In addition, Nayakshin, Kazanas, & Kallman found that the gas was thermally unstable at certain ionization parameters, which created an ambiguity in choosing solutions, and a sharp transition in temperature in the disk.

Li, Gu & Kahn (119) found a static solution that resolves the thermal instability in the gas by considering the effect of conduction, and they computed the X-ray recombination and line scattering spectrum for the conduction transition region that
forms between stable solutions in the disk. With this procedure, the unphysical, sharp transition between stable phases was eliminated. Li et al. considered ionizing continua typical of AGN, which yield three stable solutions with different temperature for a given pressure ionization parameter $\Xi$. Up to three distinct transition layers can form. The reflection and recombination spectrum of the transition regions in the 0.5–1.5 keV range was computed by considering the vertical structure of an isobaric, optically thin disk. They find that resonant scattering can be important within the transition region, depending on the local gravity and luminosity, and yielding a line spectrum which is different from that of recombination emission.

The vertical structure of an optically thick accretion disk can be obtained using the diffusion approximation, which assumes that the photon mean free path $\lambda$ is much smaller than the scale of temperature and density gradients $T/\nabla T$ and $\rho/\nabla \rho$, respectively. Adding convective heat transfer by introducing an adiabatic temperature gradient, Meyer & Meyer-Hofmeister (140) have calculated the vertical structure of an isolated accretion disk which is dominated by convection zones. Such techniques are used in the standard stellar structure equations. X-ray illumination from the central compact object suppresses convection, reduces the thermal gradients in the disk, and has a stabilization effect in the outer radii, but it also produces a convex disk that self-shadows the outer disk regions, contradicting the observed spectra, which show evidence of reprocessing (55). A semi-analytic model using a variable $\alpha$-viscosity prescription was obtained to model AGN disks and investigate its effects on the Ly edge and emission (181).

The failure of the diffusion-equation models to reproduce a concave disk that can efficiently reprocess the central X-rays may indicate that important effects were neglected. Turbulent heat transfer may be dominating, creating a vertical disk struc-
ture that is nearly isothermal. The strong turbulence occurring at the scale of the disk thickness in MHD models supports this hypothesis (141). A reliable calculation of the turbulent heat transfer in an accretion disk is needed. Thus, we prefer to use the Vršnak et al. (225) vertically-isothermal disk for the optically thick region.

The diffusion approximation is inadequate when calculating high-resolution spectra since line radiation must originate in a region where the photon mean-free path \( \lambda \) exceeds the scale of the temperature gradient, i.e. \( \lambda \gtrsim T/\nabla T \). Thus, just as for stellar atmospheres (142), an explicit radiation transfer calculation without assumption of local thermodynamic equilibrium is needed. The modeling of photoionization heating, recombination cooling, and X-ray opacities is required when the illumination energy is large compared to the energy dissipated in the disk.
Chapter 8

A Model of a Photoionized Accretion Disk

I model the X-rays reprocessed by an accretion disk in a fiducial low-mass X-ray binary system with a neutron star primary. An atmosphere, or the intermediate region between the optically thick disk and a Compton-temperature corona, is photoionized by the neutron star continuum. X-ray lines from the recombination of electrons with ions dominate the atmosphere emission and should be observable with the Chandra and XMM-Newton high-resolution spectrometers. The self-consistent disk atmosphere geometry agrees well with optical observations of these systems. At a critical depth range, the disk gas has one thermally unstable and two stable solutions. A clear difference between the model spectra exists between evaporating and condensing disk atmospheres. This difference should be observable in high-inclination X-ray binaries, or whenever the central continuum is blocked by absorbing material and the extended disk emission is not.
8.1 INTRODUCTION

The X-rays reprocessed on accretion disks to the optical, UV and X-ray bands in both X-ray binaries and AGN could be one of the best probes of accretion disk structure and the environment around compact objects. It is convenient to study an illuminated disk around neutron star low-mass X-ray binaries (LMXB) since the central continuum dominates the energetics of the disk atmosphere and accretion disk corona (ADC), and the system geometry is often determined. Measurements of the optical magnitudes and lightcurve amplitudes of LMXBs yielded estimates of the angle subtended by the disk of \( \sim 12^\circ \) and of the average fraction of X-rays reprocessed by the disk into the optical of \( \lesssim 0.1 \) (48). Models that seem to describe correctly the accretion disk interior have been unable to produce disk geometries consistent with the optical data (55). Analytical models assuming an isothermal disk show better agreement with the data, yet produce illuminated disks that are significantly thinner than expected (225).

A pure continuum incident on the gaseous disk will photoionize and heat the gas. Radiative heating sustains a \( \sim 10^7 \) K ADC and a smaller \( \sim 10^5-10^6 \) K atmosphere emitting X-ray lines on top of the optically thick disk. X-ray spectra of LMXBs during dips show strong Comptonized emission plus an excess flux at 0.65 keV consistent with the same covering fraction (37), suggesting that ADC and disks may coexist. The apparent size of the ADC and the disk may not be equal (38), since the emission mechanisms have different radial dependence. Nayakshin, Kazanas & Kallman (146) modeled the reprocessed continuum and fluorescence emission from the inner disk of AGN with an ADC, and Ko & Kallman (107) modeled X-ray spectra of disk annuli in an LMXB with ADC and normally incident radiation. Li, Gu & Kahn
8.2. MODEL ATMOSPHERE

(119) modeled the X-ray recombination and line scattering spectra of the conduction transition region present in an optically thin, isobaric disk atmosphere, considering the effect of a thermal instability.

In this chapter, I calculate the structure of an accretion disk illuminated by the grazing radiation of a neutron star in an \( LMXB \) with a Compton-temperature \( ADC \), building on a model by Raymond (173). This model includes a more accurate calculation of the vertical and radial disk structure than previous models, accounts for thermal instabilities, and incorporates recent atomic model calculations that predict X-ray spectra at a resolution superior to the \( Chandra \) and \( XMM-Newton \) spectrometers. I obtain the first self-consistent flared disk geometry that matches the disk thickness expected from optical data, and I predict that broad X-ray lines will be observable and will provide, among other diagnostics, evidence for photo-evaporation, which is central to the origin of winds (246).

8.2 MODEL ATMOSPHERE

Consider an \( LMXB \) with a \( M_\star = 1.4 \, M_\odot \) primary radiating an Eddington luminosity \( (L_\times = 10^{38.3} \, \text{erg s}^{-1}) \) bremsstrahlung continuum with \( T = 8 \, \text{keV} \). A set of fiducial system parameters for a bright \( LMXB \) is used, so application to a particular source will require using the observed X-ray continuum to improve accuracy. The maximum radius of the centrally-illuminated disk is \( 10^{11} \, \text{cm} \), so the orbital period \( \sim 1 \, \text{day} \). The minimum radius is \( 10^{8.5} \, \text{cm} \), below which the omitted effect of radiation pressure dominates.

The vertical disk atmosphere structure of each annulus is obtained by integrating the hydrostatic balance and 1-D radiation transfer equations for a slab geometry (Fig.
Figure 8.1: Side view schematic of an illuminated accretion disk and the model geometry, assuming an extended corona above the disk. To compute the ionization structure of the disk, the disk geometry (a), can be approximated by a series of gas columns which are illuminated from the top (b).

\begin{align*}
\frac{\partial P}{\partial z} &= -\frac{GM_\ast \rho z}{r^3} \quad \text{(8.1)} \\
\frac{\partial F_\nu}{\partial z} &= -\frac{\kappa_\nu F_\nu}{\sin \theta} \quad \text{(8.2)} \\
\frac{\partial F_\nu^d}{\partial z} &= -\kappa_\nu F_\nu^d \quad \text{(8.3)}
\end{align*}

while satisfying local thermal equilibrium (Eq. 8.11):

\begin{equation}
\Lambda(P, \rho, F_\nu) = 0 \quad \text{(8.4)}
\end{equation}
and ionization balance (Eq. 7.2):

\[
\text{ion formation rate} = \text{ion destruction rate}
\]  

where \( P \) is the total pressure, \( \rho \) is the density, \( F_\nu \) is the incident radiation field, \( F^\text{d}_\nu \) is the reprocessed radiation propagating down towards the disk midplane, \( z \) the vertical distance from the midplane, \( G \) the gravitational constant, \( \theta \) the grazing angle of the radiation on the disk, \( r \) the radius, and \( \kappa_\nu \) is the local absorption coefficient.

For the structure calculation only, 100 logarithmically spaced energy bins, in the range \( 1 \text{ eV} < \nu < 1000 \text{ keV} \), were used in \( F_\nu \) and \( F^\text{d}_\nu \). The reprocessed radiation propagating upwards, \( F^\text{u}_\nu \), is omitted to accelerate the computation. This is a good approximation since the radiative heating is dominated by the direct flux \( F_\nu \). The reprocessed flux \( F^\text{u}_\nu \) is calculated \textit{a posteriori} by a high-resolution spectral model (Section 8.4). The difference between cooling and heating \( \Lambda \) includes Compton scattering, bremsstrahlung cooling, photoionization heating, collisional line cooling, and recombination line cooling (Section 8.2.2). Cosmic abundances (2) are assumed. The code from Raymond (173) computes the net heating and ionization equilibrium, Compton scattering, and line scattering using escape probabilities. A new disk structure calculation simultaneously integrates Eqs. 8.1-8.3 by the Runge-Kutta method using an adaptive stepsize control routine with error estimation, and Eq. 8.4 is solved by a globally convergent Newton’s method (168). At the \textit{ADC} height \( z_{\text{cor}} \), the equilibrium \( T \) is close to the Compton temperature \( T_{\text{compton}} \), from which I begin to integrate downward until \( T < T_{\text{phot}}(r) \). The optically thick part of the disk, with temperature \( T_{\text{phot}} \), is assumed to be vertically isothermal (225). To get \( T_{\text{phot}} \), the viscous energy and illumination energy is assumed to be locally radiated as blackbody radiation. Thus, for \( z_{\text{phot}} \ll r \) and \( R_* \ll r \), Eq. 7.7 can be used with \( M_1 \equiv M_* \). The height
at which the integration ends is defined as the photosphere height $z_{ph}$. Thus, the assumption is that for $z < z_{ph}$ viscous dissipation dominates heating (Fig. 8.2).

Figure 8.2: Gas column geometry for each disk annulus. The boundary conditions on the atmospheric structure are shown.

The boundary conditions at the ADC are set to $P(z_{cor}) = \rho_{cor} k T_{compton} / \mu m_p$, $F_x(z_{cor}) = L_x / 4\pi r^2$, and $F_{\nu}^d(z_{cor}) = 0$, where $F_x \equiv \int F_{\nu} d\nu$, $k$ is the Boltzmann constant, and $\mu$ is the average particle mass in units of the proton mass $m_p$. The boundary conditions at $z_{ph}$ for $F_{\nu}$ and $F_{\nu}^d$ are set free, and the shooting method (168) is used with shooting parameter $\rho_{cor}$, which is adjusted until $P(z_{ph}) = \rho_{ph} k T_{ph} / \mu m_p$ is satisfied at the photosphere. Note $\rho_{ph}$ is the viscosity-dependent density calculated for an X-ray illuminated Shakura & Sunyaev (197) disk, where the dimensionless viscosity parameter is denoted by $\alpha$.

The shooting method consists of guessing a value of the coronal density that matches the desired pressure at the bottom of the gas column. The boundary conditions define $P_{cor}$ once $\rho_{cor}$ is chosen. Eqs. 8.1 through 8.4 are simultaneously solved.
during the integration. The temperature of the atmosphere drops as the integration proceeds. When it reaches a value below $T_{\text{phot}}$, the pressure at that point is compared to the expected pressure of the isothermal disk at that height. If it does not match to better than $\sim 1\%$, the integration is repeated with a new estimate of the coronal density. While it is not clear that photoionization will cease to be important for temperatures less than $T_{\text{phot}}$, such zones emit negligible X-ray fluxes if $r \gtrsim 10^9$ cm. The shooting method yields a repeatable structure for each disk annulus, satisfying hydrostatic equilibrium to $\lesssim 1\%$ and thermal balance to $\lesssim 0.01\%$.

An important feature of this model is that the incident radiation is allowed to modify the disk atmosphere geometry, such that the heating and expansion of the atmosphere resulting from illumination are used to calculate the height profile of the atmosphere as a function of radius. This feedback between the radiative heating and the disk geometry is shown in Fig. 8.3. The atmospheric height is used to derive the input grazing angle of the radiation for the next model iteration. This contrasts with calculating the grazing angle using the pressure scale height of the optically thick disk (225), which is in general well below the photoionized atmosphere, and which underestimates the grazing angle and the line intensities by an order of magnitude. To get $T_{\text{phot}}$ self-consistently from Eq. 7.7, the equation

$$\theta(r) = \arctan\left(\frac{dz_{\text{atm}}}{dr}\right) - \arctan\left(\frac{z_{\text{atm}}}{r}\right) + \arctan\left(\frac{R_\star}{r}\right)$$

(8.6)
is needed, where $z_{\text{atm}}(r)$ is defined as the height where the frequency-integrated grazing flux $F_x$ is attenuated by $e^{-1}$. The term $\propto R_\star/r$ is neglected, which is valid for $r \gtrsim 10^{8.5}$ cm. As discussed above, $\theta(r)$ is calculated iteratively from Eq. 8.6. After an initial guess for $z_{\text{atm}}(r)$, it is re-calculated from the newly obtained disk structure. The iteration is stopped after $\theta(r)$ and $T_{\text{phot}}$ converge to $\lesssim 10\%$. Note $F_x \to 0$ for $z \lesssim z_{\text{phot}}$, where the disk blackbody flux $F_{bb}(T_{\text{phot}})$ takes over. Since $z_{\text{atm}}$ is not
physically determined, it must be defined *ad hoc*, but it is bound by $z_{\text{atm}} > z_{\text{phot}}$. A second model calculated $\theta(r)$ from $z_{\text{atm}} = z_{\text{phot}}$, which was used to estimate the systematic errors of the 1-D radiation transfer calculation.

Figure 8.3: Schematic of the feedback between radiative heating and disk geometry. The heated atmosphere expands and collects more radiation, reaching equilibrium at $\sim 10$ times its initial volume.

### 8.2.1 The Choice of Assumptions

The validity of the assumptions is reviewed, both for the the model presented here and for accretion disk models in the astrophysical literature.

For modeling X-ray line emission from the disk atmosphere, the commonly used assumptions of *LTE* and the diffusion approximation will not hold. In addition, assuming a constant density in the vertical direction will be inadequate, since the hydrostatic equilibration time is small or comparable to other relevant timescales, and the line emission is highly sensitive to the vertical ionization structure. This is
especially true for recombination emission, in which each ion has a characteristic set of line energies, and it contrasts with fluorescent line energies, which vary little with ionization state until the atom is almost fully ionized. Yet, even fluorescence emission can be affected by a Compton-thick, fully ionized gas above it (146).

Hydrostatic equilibrium, thermal equilibrium, and ionization equilibrium should be good assumptions in a time averaged sense. Deviations from hydrostatic equilibrium are smoothed in the $t_{\text{hydro}} = z_{\text{atm}} / c_s$ timescale, where $c_s$ is the sound speed. Material from the disk moves radially within the viscous timescale $t_{\text{visc}} \sim r^2 / \alpha z_{\text{atm}} c_s > t_{\text{hydro}}$ (62), so the gas can reach hydrostatic equilibrium before it flows inward (i.e., the radial accretion velocity is always subsonic). However, the Keplerian orbital velocity $v_k \gg c_s$ is highly supersonic. If the gas flow in the corotating frame of the gas is also supersonic, then shocks would collisionally ionize and heat the gas. In such a case, the observed spectrum of the disk would significantly deviate from a photo-ionized gas. Since such effects have not been observed in the X-ray line emission (section 9), the hydrostatic equilibrium assumption is verified, at least for the line-emitting region. The thermalization and ionization equilibrium timescale of the atmosphere is driven by the recombination timescale

$$t_{\text{th}} = t_{\text{rec}} \approx 0.3 \left( \frac{T_5^{1/2}}{n_{14} Z^2} \right) \text{ sec}$$

(8.7)

where $T_5$ is the temperature in units of $10^5$ K, $n_{14}$ is the density in units of $10^{14}$ particles cm$^{-3}$, and $Z$ is the ion atomic number (175). The photoionization timescale is shorter than the recombination timescale, since it is a photon-ion interaction, and not an electron-ion interaction (section 7.3.1). The Coulomb collision relaxation timescale between electrons and ions $t_{\text{ep}}$ is slower than between identical particles, and is (204)

$$t_{\text{ep}} \approx 10 \frac{T_{3/2}^3}{n_e} \text{ sec} = 3 \times 10^{-6} \frac{T_5^{3/2}}{n_{14}} \text{ sec.}$$

(8.8)
Electron-electron relaxation is $\sim m_p/m_e$ times faster, and proton-proton relaxation is $\sim \sqrt{m_p/m_e}$ times faster. For the hot corona at the outer disk at $T \sim 10^7$ K and $n_e \sim 10^{11}$ cm$^{-3}$ (from the coronal structure in Fig. 8.8), the relaxation timescale is $t_{ep} \sim 3$ sec. The fully ionized gas in the corona which is near the Compton temperature has a thermal timescale of $t_{th} = t_{\text{compton}} = 10r_{10}^2L_{38}^{-1}$ sec, where $r_{10}$ is the disk radius in units of $10^{10}$ cm, and $L_{38}$ is the X-ray luminosity in units of $10^{38}$ ergs$^{-1}$ (175). Thus, thermalization in the disk atmosphere and corona is driven by the ionization timescales, since the Coulomb relaxation times are comparatively fast due to the large densities. Thermal and ionization equilibrium occur faster than hydrostatic equilibrium. Since $t_{th} < t_{\text{hydro}}$, any observed luminosity fluctuations with timescales $t_{\text{flux}}$ such that $t_{th} < t_{\text{flux}} < t_{\text{hydro}}$ will take the gas outside hydrostatic equilibrium. Integrated spectral observations on timescales $t \gg t_{\text{flux}}$ will, of course, be unable to observe this effect.

The radiation transfer is complex, and the assumptions used to simplify calculations could be problematic. In particular, by dividing the disk atmosphere into annular zones with a given vertical gas column, our 1-D radiation transfer calculation assumes that 1) the primary continuum is not absorbed before reaching the top of the column, 2) the radiation in the column propagates from top to bottom at a given grazing angle, and 3) there is no significant radiative coupling from one disk annulus to another, which is used to justify the slab approximation. The above assumptions are inadequate if the column height is comparable to the disk radius, or if the photon mean free path in the gas column is many times the local radius. Thus, future 2-D calculations will result in better bookkeeping of photons, a more accurate structure, and a more reliable X-ray line spectrum.

The correct calculation of line transfer in the gas is also a concern, since the
disk atmosphere is optically thick in the lines. Line transfer is complicated by the
Keplerian velocity shear, which has to be taken into account for a given viewing
angle (144). The escape-probability approximation used to calculate line transfer in
the disk may also be inadequate because of the large optical depths.

Our calculations show that the proper treatment of a thermal instability (59; 112) and the effect of conduction to resolve it affect the spectrum significantly (250)
(section 8.3). A two-phase gas could form, with clouds of an unknown size distribution
and with undetermined dynamics of evaporation and condensation (13), with each
phase having distinct ionization parameter and opacity. The instability is sensitive
to 1) the metal abundances, 2) the continuum shape (80), and 3) the atomic kinetics
(187). Finally, the inclusion of radiation pressure in high luminosity systems and
small radii is advisable (171).

The local viscous energy dissipation rate per unit volume in the disk atmosphere
can be included in Eq. 8.4 with the form (197; 46)

\[ Q_{\text{visc}} = \frac{3}{2} \Omega \alpha P \]  \hspace{1cm} (8.9)

where \( \Omega \) is the Keplerian angular velocity, \( \alpha \) is the viscosity parameter, and \( P \) is
the local pressure. Eq. 8.9 is an extension of the \( \alpha \)-disk model (where the viscous
dissipation is vertically averaged), assumes the local validity of the \( \alpha \) prescription, and
is untested. Fortunately, our numerical modeling indicates that the viscosity term is
negligible in most regions of the disk atmosphere except for the inner disk if \( \alpha \sim 1 \)
(in particular, near the Compton-temperature corona). Its effect is to enhance a
thermal instability between \( 10^6 \) and \( 10^7 \) K. Vertically stratified \( MHD \) models (141),
although inconclusive, owing to the uncertain effect of boundary conditions, show
that the viscous dissipation drops rapidly \( \gtrsim 2 \) pressure scale heights away from the
disk midplane, providing evidence against Eq. 8.9. The disk atmosphere is always a few scale heights above the midplane. Therefore, we choose not to include this term in our models. Equation 8.9 has been applied in the optically thick regions of the disk (55) and in the disk atmosphere (180). Other forms for the local dissipation that reduce to the \( \alpha \)-disk have been used (140).

### 8.2.2 Thermal Equilibrium

The terms included in the thermal equilibrium equation, solved with the Raymond (173) photoionized plasma code, are reviewed. Thermal equilibrium is enforced at each zone in the disk atmosphere. The explicit form of the thermal equilibrium condition, Eq. 8.4, is

\[
\text{Compton net heating + photoionization heating} = \text{bremsstrahlung cooling + recombination cooling + collisional cooling,}
\]

which corresponds to (74)

\[
\int F_\nu \left( n_e \Gamma_\nu^{\text{com}} + \sum_{z,i} n_{z,i} \Gamma_\nu^{\text{phot}} \right) d\nu = n_e \sum_{z,i} n_{z,i} \left( \Lambda_z^{\text{brem}} + \Lambda_{z,i}^{\text{rec}} + \Lambda_{z,i}^{\text{col}} \right)
\]  

in units of \( \text{erg s}^{-1} \text{ cm}^{-3} \), where the rates for each process and other dependencies are included in the group of coefficients \( \left( \Gamma_\nu^{\text{com}}, \Gamma_\nu^{\text{phot}}, \Lambda_z^{\text{brem}}, \Lambda_{z,i}^{\text{rec}}, \Lambda_{z,i}^{\text{col}} \right) \), \( n_e \) is the electron number density, \( n_{z,i} \) is the \( Z^{+i} \) ion density with atomic number \( Z \), and the sums are performed over all abundant ions. The radiative heating is directly proportional the net flux \( F_\nu \) (in units of photons \( s^{-1} \text{ cm}^{-2} \text{ keV}^{-1} \)), and the density. Cooling processes, which originate from electron-ion interactions, are proportional to the square of the density and are in general dependent on the electron temperature \( T \). The coefficients in Eq. 8.11 can be obtained from Halpern & Grindlay (74), and some, such as the
recombination coefficients, are very dependent on the available atomic data. A list
of the data used for the coefficients in the model, and a list of the processes and
transitions included in the calculations can be found in Raymond (173). Recombina-
tion cooling included both radiative recombination and dielectronic recombination.
Collisional cooling included cooling due to line emission and collisional ionization.

The ions of H, He, C, N, O, Ne, Mg, Si, S, Ar, Ca, and Fe were included in the
thermal equilibrium (Eq. 8.11) and ionization balance (Eq. 7.2) equations.

For a fully ionized gas, such as the hot corona above the accretion disk, Compton
heating and inverse-Compton cooling dominate Eq. 8.11. The Compton net heating
term can be positive or negative, since the transfer of energy between the photons and
the electron gas depends on the electron temperature and the shape of the ionizing
spectrum. In such cases, the equilibrium temperature is (124)

\[
T_{\text{compton}} = \frac{\hbar \int \nu^2 F_\nu d\nu}{4k \int \nu F_\nu d\nu}
\]  \hspace{1cm} (8.12)

where \( k \) and \( \hbar \) are the Boltzmann and the Planck constants, respectively. The Compton
temperature \( T_{\text{compton}} \) is determined uniquely by the shape of the ionizing continuum. For an 8 keV bremsstrahlung spectrum, \( T_{\text{compton}} \sim 2 \times 10^7 \) K.

8.3 Thermal Instability in Photoionized Gases

Irradiated gas is subject to a thermal instability for temperatures in the \( 10^5-10^6 \)
K range (20; 60), suppressing X-ray line emission in that regime. The Field (59)
stability criterion, together with the plasma codes (Sections 7.3.4 and 8.2.2), indicate
that a photoionized gas may become unstable when recombination cooling of H-like
and He-like ions is important. The disk atmosphere exhibits a thermally unstable
region, which has direct consequences for the disk structure and X-ray spectrum.

To clarify the nature of the thermal instability, consider a contour map of the
net heating in the gas (Fig. 8.4). Because the photoionized gas is externally heated,
and the net heating rate depends on the state variables and ionization of the gas, this
system is quite peculiar. The thermal equilibrium locus, where the net heating is zero,
is displayed in Fig. 8.4 as a function of temperature and ionization parameter. The
neighboring contours correspond to net cooling at the left of the equilibrium locus,
while the contours at the right of the equilibrium locus correspond to net heating. The
thermal equilibrium locus has a characteristic S-shape in the ionization parameters
of interest, so it is denoted as the \( S \)-curve.

To determine whether the thermal equilibrium solutions are thermally stable,
consider small temperature perturbations around equilibrium. Because isobars have
constant \( \Xi \), an isobaric temperature perturbation will be a vertical displacement in
Fig. 8.4. Starting from a slightly perturbed temperature, the gas will heat or cool
depending in which region in Fig. 8.4 the perturbed gas is located. From these
considerations, the segments of the thermal equilibrium curve with positive slope will
be thermally stable, while those with negative slope will be unstable \((59)\). In the
unstable part of the equilibrium locus, small temperature perturbations will cause a
thermal runaway to either stable branch at constant \( \Xi \).

Within a range of pressure ionization parameters \( \Xi = P_{\text{rad}}/P_{\text{gas}} \) (Section 7.3.3),
thermal equilibrium is achieved by three distinct temperatures, only two of which
are stable to perturbations in \( T \). The temperature and ionization parameter ranges
where the instability occurs depend on the metal abundances and the local radiation
spectrum \((80)\). The calculated thermal equilibrium locus for the disk atmosphere
is shown in Fig. 8.5. Previous spectral studies of illuminated accretion disks in LMXBs had not explicitly selected the stable solutions (107), a choice which affects X-ray production.

The instability implies a large temperature gradient as the gas is forced to move between stable branches, requiring the formation of a transition region whose size may be determined by electron heat conduction, convection, or turbulence, depending on which dominates the heat transfer. For simplicity, emission from the transition region is neglected. Upon calculation of the Field length $\lambda_F$, the lengthscale below
which conduction dominates thermal equilibrium (13), I estimated that conduction forms a transition layer \( \sim 10^{-2} \) times thinner than the size of the X-ray emitting zones. Nevertheless, X-ray line emission from the neglected conduction region may not be negligible in all cases (119). The ionization parameters in the transition region do not exist elsewhere in the gas, which does enhance its influence on the spectrum (119). The importance of the transition region depends critically on the shape of the instability curve, which varies with the continuum shape (hardness in particular), and on the metal abundances (80). The instability curve is also subject to uncertainties in the atomic data (187), and variations in the calculated curves exist among photoionized plasma codes, which remain to be quantified.

Conduction determines the dynamic behaviour of the gas. The dynamics will allow us to attach a physical interpretation to the chosen solutions from Fig. 8.5. The static conduction solution produces a transition region which splits the instability \( \Xi \)-range in two (Fig. 8.5). This solution (250) takes the low-\( T \) stable branch at \( \Xi < \Xi_{\text{stat}} \), and the high-\( T \) stable branch at \( \Xi > \Xi_{\text{stat}} \), separated by the transition layer at \( \Xi_{\text{stat}} \). A transition layer located away from \( \Xi_{\text{stat}} \) will dynamically approach \( \Xi_{\text{stat}} \) by a conduction driven mass flow, as shown in Fig. 8.6. A transition layer at \( \Xi_{\text{evap}} > \Xi_{\text{stat}} \) is moving towards lower \( \Xi \), and the gas is evaporating from the low-\( T \) branch to the high-\( T \) branch. A transition layer at \( \Xi_{\text{cond}} < \Xi_{\text{stat}} \) is moving towards higher \( \Xi \), with the gas condensing to the low-\( T \) branch (250; 119).

The disk structure for both condensing and evaporating solutions is computed. I assume a steady state, condensing or evaporating mass flow through the transition layer at \( \Xi_{\text{cond}} \) or \( \Xi_{\text{evap}} \), respectively. The static conduction solution is an intermediate case of the latter extreme cases. A single-valued \( T(\Xi) \) is used, since a two-phase solution would be buoyantly unstable, making the denser (colder) gas sink. The evap-
orating disk corresponds to the low-$T$ branch, while the condensing disk corresponds to the high-$T$ branch (Fig. 8.5). This introduces spectral differences (section 8.6).

I do not know from first principles whether the disk atmosphere is evaporating, condensing, or static. The speed of the conduction mass flow is estimated to be
\[ v_{\text{cond}} = 2\kappa T/3 P_{\text{gas}} \lambda_F, \]
where \( \kappa \) is the Spitzer (204) conductivity (134). A conduction mass flow speed \( v_{\text{cond}} = 1-2 \times 10^{-2} \) times the local sound speed is obtained. Thus, the phase dynamics will depend on the subsonic \( (v \gtrsim v_{\text{cond}}) \) flow patterns in the disk atmosphere, and these flows will in part determine the evaporation or condensation rates, together with the boundary conditions on mass flow.

If the disk is in a steady state of evaporation or condensation, the implied mass flow can have an effect on the global mass budget, due to mass conservation. In the one dimensional case (250; 119), steady state evaporation (with the transition region at a fixed height) implies a mass flow from the cold to the hot state, and an overall mass loss in the system. Thus, the evaporating solution is associated with a steady state mass loss or disk wind. Similarly, the steady state condensing solution implies a mass gain. Otherwise, the transition region will move to the static solution, as it is shown in Fig. 8.6 (250; 119).

It should be noted that a thermal instability due to Compton heating and bremsstrahlung cooling can ensue between \( 10^6 \) and \( 10^7 \) K if the X-ray ionizing spectrum extends well above \( \sim 10 \) keV (112). For the model discussed here, which assumes an 8 keV bremsstrahlung ionizing spectrum, this instability is suppressed (80). Nevertheless, some LMXBs do have such hard spectra (Section 7.2.2). In those cases, re-modeling with a harder spectrum will be necessary, and the issue of resolving an additional instability regime will again be encountered. Thermal equilibrium loci
with the shape of a double S-curve have been calculated for such hard spectra (147),
which allow the existence of a three-phase gas.

As mentioned above, gas dynamics which are not included in the disk model can
have an impact on the gas phase. The only physical mechanism known to transport
the necessary angular momentum for disk accretion involves a magneto-rotational
instability (MRI) which drives turbulent flow in the disk (7). These turbulent flows
are nearly supersonic in the core of the disk, where most of the mass is accreted.
Enhanced heat transfer rates due to this turbulent flow could quench the thermal
instability and affect the disk structure. Turbulent heat transfer rates can be orders
of magnitude larger than the saturated conduction heat transfer rate. However, it is
not known whether such turbulent motions will also be present in the disk atmosphere,
which is several scale heights above the disk core and has a density which is orders of
magnitude smaller (Section 8.5). A decline in the viscous $\alpha$ parameter with vertical
disk height was obtained with local MHD models, and an enhanced ratio of the
magnetic pressure to the gas pressure with increasing height (141). The MRI also
favors the assumption of vertical isothermality in the optically thick disk.

8.4 SPECTRAL MODELING

With the disk structure $\rho(r, z)$, $T(r, z)$ and ion abundances $f_{z,i}(r, z)$, the X-ray line
emission from the disk atmosphere is modeled by using HULLAC (Hebrew University/Lawrence Livermore Atomic Code, Klapisch et al. (105)). The code calculates
the atomic structure and transition rates of radiative recombination ($RR$) and ra-
diative recombination continuum ($RRC$) for the H-like and He-like ions of C, N, O,
Ne, Mg, Si, S, Ar, Ca, and Fe, as well as the Fe L shell ions. Fluorescence emission,
which is prominent for high-\(Z\) ions such as those of Fe, is omitted in these calculations, as well as resonant scattering, an additional source of line emission. Li, Gu 
& Kahn (119) have found that resonant scattering can be important in an accretion disk atmosphere, and that it will produce a spectrum which is different from radiative recombination. The recombination emissivities in this model are calculated as described in section 8.4.1.

The spectrum for each annulus is added to obtain the disk spectrum. Each annulus consists of a grid of zones in the vertical \(\hat{z}\) direction, and \(T, \rho\) and \(f_{z,i+1}\) for each zone are used to calculate the \(RR\) and \(RRC\) emissivities. The radiation is propagated outwards at inclination angle \(i\), including the continuum opacity of all zones above, thus accounting for the optical depth of the atmosphere. Compton scattering is not included in the calculated spectrum because it is negligible in the vertical direction in the region where the line emission is formed. The spectrum is Doppler broadened by the projected local Keplerian velocity, assuming azimuthal symmetry.

### 8.4.1 Radiative Recombination Emission

I describe the numerical calculation of the radiative recombination emission emissivity, including both radiative recombination lines and radiative recombination continua. A similar method for an optically thin gas in the photoionized wind of a High Mass X-ray Binary \(HMXB\) was described by Sako et al. (183), and some of their notation will be followed here.

Consider an infinitesimal volume \(dV\) at which a single ionization parameter \(\xi\), temperature \(T\), electron density \(n_e\), and elemental abundances \(A_z\) describe the state of
a gas. A relationship $\xi(T)$ can be found from thermal balance and ionization equilibrium, for a given ionizing spectrum $F_\nu$ (Section 7.3.3). In the radiative recombination process (Section 7.3.1)

$$Z^{+(i+1)} + e^- \rightarrow Z^{+i} + h\nu_{\text{RRC}} \quad (8.13)$$

an electron recombines with an ion of atomic number $Z$ and net charge $+(i + 1)$, assumed in its ground state, producing a new ion with net charge $+i$, which might be excited. The radiative recombination continuum photon has energy

$$E_x = h\nu_{\text{RRC}} = \chi + KE_e, \quad (8.14)$$

where $\chi$ is the ionization potential of ion $Z^{+i}$, and $KE_e$ is the initial kinetic energy of the electron, assumed to be on a Maxwell distribution with temperature $T$. The radiative recombination rate in units of $s^{-1}$ is

$$\Gamma_{\text{RR}} = n_e n_{z,i+1} \alpha_{\text{RR}} dV \quad (8.15)$$

where $n_{z,i+1}$ is the $Z^{+(i+1)}$ ion number density, Eq. 8.15 defines $\alpha_{\text{RR}}$, the radiative recombination rate coefficient in units of cm$^3$ s$^{-1}$. Note $\alpha_{\text{RR}}$ depends on $Z$ and $i$.

**Radiative Recombination Lines**

After recombination, a fraction $\eta_{u\rightarrow l}$ of the $Z^{+i}$ ions produce a radiative cascade photon by an electronic transition from upper level $u$ to lower level $l$ (Section 7.3.1). The line luminosity of photons from this transition in units of erg s$^{-1}$ is

$$dL_{u\rightarrow l} = n_e n_{z,i+1} E_{u\rightarrow l} \eta_{u\rightarrow l} \alpha_{\text{RR}} dV \quad (8.16)$$

where $E_{u\rightarrow l}$ is the transition energy in ergs. To create a synthetic spectrum, the line luminosities $dL_{u\rightarrow l}$ at energies $E_{u\rightarrow l}$ in the X-ray band are added for all the levels.
8.4. SPECTRAL MODELING

\( u, l, \) and all the ions \( Z^{+(i+1)} \) which are abundant in the gas. Notice the recombination emission of the \( Z^{+i} \) ion depends on the number density of the \( Z^{+(i+1)} \) ion.

For computational purposes, various quantities from Eq. 8.16 can be defined. The specific line power

\[
S_{u \rightarrow l} \equiv \eta_{u \rightarrow l} \alpha_{RR}, \tag{8.17}
\]

in units of \( \text{cm}^3 \text{s}^{-1} \), is the photon emission rate per \( Z^{+(i+1)} \) ion, per unit electron density. The \( Z^{+(i+1)} \) ion was assumed to be in its ground state before recombining into \( Z^{+i} \). The population fraction \( f_u \) of each level of the \( Z^{+i} \) ion is computed explicitly, and \( S_{u \rightarrow l} \) is obtained by equating the matrix of the photon emission rates per ion,

\[
n_e S_{u \rightarrow l} = f_u A_{u \rightarrow l} \tag{8.18}
\]

where \( A_{u \rightarrow l} \) is the rate of spontaneous decay for \( Z^{+i} \). After solving \( S_{u \rightarrow l} \) for a grid of temperatures, it is fit to a power law

\[
S_{u \rightarrow l} = C_{u \rightarrow l} T^{-\gamma_{u \rightarrow l}} \tag{8.19}
\]

where the exponent \( \gamma_{u \rightarrow l} \) is typically 0.6–0.8. The number density of \( Z^{+(i+1)} \) is calculated with

\[
n_{z,i+1} = n_H A_z f_{z,i+1} \tag{8.20}
\]

where \( n_H \) is the proton density, \( A_z \) is the fractional abundance of element \( Z \) relative to \( H \), and \( f_{z,i+1} \) is the fractional abundance of the \( Z^{+(i+1)} \) ion relative to all the \( Z \) ions. The differential emission measure for \( Z^{+(i+1)} \) is defined as

\[
d(EM_{z,i+1}) \equiv n_e n_{z,i+1} dV, \tag{8.21}
\]

in units of \( \text{cm}^{-3} \). The line luminosity in Eq. 8.16 can therefore be re-written as

\[
dL_{u \rightarrow l} = E_{u \rightarrow l} S_{u \rightarrow l} d(EM_{z,i+1}). \tag{8.22}
\]
If the emission measure is defined as \( d(EM) \equiv n_e^2 dV \), then \( dL_{\nu,t} = P_{\nu,t} d(EM) \), where \( P_{\nu,t} \) is defined as the line power, with units of erg cm\(^3\) s\(^{-1}\). The emission measure is useful for calculating the luminosity of an optically thin gas. Since the accretion disk atmosphere does have some optical depth, \( d(EM) \) and \( d(EM_{z,t+1}) \) will only be used to track the regions where the emission originates.

**Radiative Recombination Continuum**

To calculate the shape and luminosity of the \( RRC \), a Maxwell thermal distribution, the photoionization cross sections, and the Milne relation were used. The monochromatic version of the \( RR \) coefficient in Eq. 8.15, for electrons with velocities between \( v \) and \( v + dv \) is

\[
\alpha_{RR,\nu} = \sigma_{RR,\nu} v f_v dv,
\]

where \( \sigma_{RR,\nu} \) is the \( RR \) cross section of ion \( Z^{+(i+1)} \), and the number of electrons in that velocity range is \( f_v dv \), which is assumed to be given by the Maxwellian distribution

\[
f_v = \left( \frac{2}{\pi} \right)^{1/2} \left( \frac{m}{kT} \right)^{3/2} v^2 e^{-mv^2/2kT},
\]

(8.24)

where \( m \) is the electron mass. Thus, the monochromatic \( RRC \) emissivity of \( Z^{+i} \) for thermal electrons is

\[
j_\nu = n_e n_{z,t+1} E_x \sigma_{RR,\nu} v f_v \frac{dv}{dE_x},
\]

(8.25)

in units of erg cm\(^{-3}\) s\(^{-1}\) erg\(^{-1}\). Because radiative recombination is the inverse process of photoionization, a relationship between their cross sections can be derived by equating their transition rates obtained from Fermi’s golden rule (186). Detailed balance yields a cross section ratio proportional to the ratio of the density of final states for each reaction. For recombination and photoionization, this is the Milne relation

\[
\sigma_{RR,\nu} = \frac{g_i}{g_{i+1} \left( \frac{E_x}{mc\nu} \right)} \sigma_{PE,\nu},
\]

(8.26)
8.4. SPECTRAL MODELING

where $\sigma_{PE,\nu}$ is the photoionization cross section for the valence electron of $Z^+$, and $g_i, g_{i+1}$ are the statistical weights of the energy levels of ions $Z^+$ and $Z^{+i+1}$, respectively. Note $g = 2J + 1$, for total angular momentum quantum number $J$. From Eqs. 8.14, and 8.24–8.26, one can derive the RRC emissivity

$$j_\nu = \left( \frac{2}{\pi} \right)^{1/2} n_e n_{z,i+1} \frac{g_i}{g_{i+1}} c \sigma_{PE,\nu} \left( \frac{E_x^2}{m c^2 k T} \right)^{3/2} e^{-(E_x-\chi)/kT}$$

which is in the same units as Eq. 8.25. The photoionization cross sections are taken from Salomon, Hubble, and Scofield (185):

$$\sigma_{PE,\nu} = 10^{-18} n' \frac{R_y}{\chi'} \exp \left[ \sum_{q=0}^{3} a_q \left( \ln \frac{E_x}{\chi'} \right)^q \right]$$

in units of cm$^2$, where the four-element $a_q$ vector and $\chi'$ are fitting parameters, and $R_y \equiv 13.6$ eV. Note $\chi' \sim \chi$. The constant $n'$ is a function of various occupancy numbers and statistical weights.

8.4.2 CONTINUUM OPACITY

In the disk atmosphere, the recombination emission is partially absorbed by the ionized gas above it. Each ionization zone in the gas column in Fig. 8.2 can be denoted by an index $j = 1...N$, starting from the top zone. If a recombination emission net flux $F_{\nu,j}$ is produced in each zone $j$ of height $h_j$, then the total flux for the column is

$$F_\nu = \sum_{j=1}^{N} F_{\nu,j} \exp \left[ \frac{1}{\cos i} \sum_{m<i} h_m \kappa_{\nu,m} \right]$$

where $i$ is the inclination angle of the observer in reference to the disk midplane normal, and $\kappa_{\nu,m}$ is the continuum opacity of the $m$th zone,

$$\kappa_{\nu,m} = \sigma_T n_{e,m} + \sum_{z,k} \sigma_{\nu,z,k} n_{z,k,m} ,$$
where \( n_{z,k,m} \) is the number density of each ion \( Z^{+k} \) in the \( m \)th zone, \( n_{e,m} \) is the electron density, \( \sigma_T \) is the Thomson cross section, and \( \sigma_{\nu,z,k} \) is the photo-electric absorption cross section of ion \( Z^{+k} \), given by

\[
\sigma_{\nu,z,k} = \sum_{e=1}^{z-k} \sigma_{PE,\nu,z,k,e}
\]  

(8.31)

where the photoionization cross section \( \sigma_{PE,\nu,z,k,e} \) for each electron \( e \) in the ion \( Z^{+k} \) is given by Eq. 8.28, and the cross sections for all \( Z - k \) electrons were added. This contrasts with the case of recombination, in which only the \( \sigma_{PE,\nu} \) for the valence electron was needed. The model atmosphere was optically thin; i.e., the continuum optical depth \( \tau_{\nu} = \sum_{j=1}^{N} h_{j} K_{\nu,j} \ll 1 \).

The flaring geometry of the disk atmosphere (see next section) is not taken into account in Eq. 8.29. Disk flaring will result in opacities that are larger than those in Eq. 8.29 at inclinations \( i > 75-80^\circ \), since the disk atmosphere subtends an angle of \( \arctan \left( \frac{z_{\text{atm}}}{r} \right) = 10-15^\circ \).

### 8.5 Disk Structure Results

The self-consistent disk is thicker than would be expected from the local pressure scale height \( H_P \) alone. I find \( H_P < z_{\text{phot}} < z_{\text{atm}} \). Both the modeled photosphere and atmosphere height, \( z_{\text{phot}} \) and \( z_{\text{atm}} \), are fitted with \( C r^{n} \) (\( r \) in cm), with fit parameters \( C \) and \( n \). The fitted parameters are \( C_{\text{phot}} = 2.4 \pm 0.4^{+0.9}_{-1.9} \times 10^{-3} \) cm\(^{1-n_{\text{phot}}} \), \( n_{\text{phot}} = 1.14^{+0.06}_{-0.04} \), \( C_{\text{atm}} = 1.0 \pm 0.1^{+0.2}_{-0.05} \times 10^{-3} \) cm\(^{1-n_{\text{atm}}} \), \( n_{\text{atm}} = 1.21 \pm 0.01 \). The above fits imply \( z_{\text{phot}} \sim 3 \) to 4 \( H_P \) (depending on radius) and \( z_{\text{atm}} \sim 7^{+2}_{-4} \) to \( 8^{+3}_{-2} \) \( H_P \). Estimated systematics are shown, if significant. Vrtilek et al. (225) estimated \( n_{\text{atm}} = 9/7 = 1.29 \), but in spite of the steeper radial dependence, the Vrtilek disk is thinner, and it equals