Is there a copy of you reading this article? A person who is not you but who lives on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets? The life of this person has been identical to yours in every respect. But perhaps he or she now decides to put down this article without finishing it, while you read on.

The idea of such an alter ego seems strange and implausible, but it looks as if we will just have to live with it, because it is supported by astronomical observations. The simplest and most popular cosmological model today predicts that you have a twin in a galaxy about 10 to the 10^28 meters from here. This distance is so large that it is beyond astronomical, but that does not make your doppelgänger any less real. The estimate is derived from elementary probability and does not even assume speculative modern physics, merely that space is infinite (or at least sufficiently large) in size and almost uniformly filled with matter, as observations indicate. In infinite space, even the most unlikely events must take place somewhere. There are infinitely many other inhabited planets, including not just one but infinitely many that have people with the same appearance, name and memories as you, who play out every possible permutation of your life choices.

You will probably never see your other selves. The farthest you can observe is the distance that light has been able to travel during the 14 billion years since the big bang expansion began. The most distant visible objects are now about 4 × 10^26 meters away—a distance that defines our observable universe, also called our Hubble volume, our horizon volume or simply our universe. Likewise, the universes of your other selves are spheres of the same size centered on their planets. They are the most straightforward example of parallel universes. Each universe is merely a small part of a larger “multiverse.”

By this very definition of “universe,” one might expect the notion of a multiverse to be forever in the domain of metaphysics. Yet the borderline between physics and metaphysics is defined by whether a theory is experimentally testable, not by whether it is weird or involves unobservable entities. The frontiers of physics have gradually expanded to incorporate ever more abstract (and once metaphysical) concepts such as a round Earth, invisible electromagnetic fields, time slowdown at high speeds, quantum superpositions, curved space, and black holes. Over the past several years the concept of a multiverse has joined this list. It is grounded in well-tested theories such as relativity and quantum mechanics, and it fulfills both of the basic criteria...
of an empirical science: it makes predictions, and it can be falsified. Scientists have discussed as many as four distinct types of parallel universes. The key question is not whether the multiverse exists but rather how many levels it has.

**Level I: Beyond Our Cosmic Horizon**

The parallel universes of your alter egos constitute the Level I multiverse. It is the least controversial type. We all accept the existence of things that we cannot see but could see if we moved to a different vantage point or merely waited, like people watching for ships to come over the horizon. Objects beyond the cosmic horizon have a similar status. The observable universe grows by a light-year every year as light from farther away has time to reach us. An infinity lies out there, waiting to be seen. You will probably die long before your alter egos can observe them through a sufficiently powerful telescope.

If anything, the Level I multiverse sounds trivially obvious. How could space not be infinite? Is there a sign somewhere saying “Space Ends Here—Mind the Gap”? If so, what lies beyond it? In fact, Einstein’s theory of gravity calls this intuition into question. Space could be finite if it has a convex curvature or an unusual topology (that is, interconnectedness). A spherical, doughnut-shaped or pretzel-shaped universe would have a limited volume and no edges. The cosmic microwave background radiation allows sensitive tests of such scenarios [see “Is Space Finite?” by Jean-Pierre Luminet, Glenn D. Starkman and Jefrey R. Weeks; Scientific American, April 1999]. So far, however, the evidence is against them. Infinite models fit the data, and strong limits have been placed on the alternatives.

Another possibility is that space is infinite but matter is confined to a finite region around us—the historically popular “island universe” model. In a variant on this model, matter thickens on large scales in a fractal pattern. In both cases, almost all universes in the Level I multiverse would be empty and dead. But recent observations of the three-dimensional galaxy distribution and the microwave background have shown that the arrangement of matter gives way to void uniformity on large scales, with no coherent structures larger than about 10^33 meters. Assuming that this pattern continues, space beyond our observable universe teems with galaxies, stars and planets.

Observers living in Level I parallel universes experience the same laws of physics as we do but with different initial conditions. According to current theories, processes early in the big bang spread matter around with a degree of randomness, generating all possible arrangements with nonzero probability. Cosmologists assume that our universe, with almost uniform distribution of matter and initial density fluctuations of one part in 100,000, is a fairly typical one (at least among those that contain observers). That assumption underlies the estimate that your closest identical copy is 10 to the 118th meters away. About 10 to the 10^9 meters away, there should be a sphere of radius 100 light-years identical to the one centered here, so all perceptions that we have during the next century will be identical to those of our counterparts over there. About 10 to the 10^30 meters away should be an entire Hubble volume identical to ours.

These are extremely conservative estimates, derived simply by counting all possible quantum states that a Hubble volume can have if it is no hotter than 10^8 kelvins. One way to do the calculation is to ask how many protons could be packed into a Hubble volume at that temperature. The answer is 10^115 protons. Each of those particles may or may not, in fact, be present, which makes for 2 to the 10^115 possible arrangements of protons. A box containing that many Hubbles volumes exhausts all the possibilities. If you round off the numbers, such a box is about 10 to the 10^118 meters across. Beyond that box, universes—including ours—must repeat. Roughly the same number could be derived by using thermodynamic or quantum-gravitational estimates of the total information content of the universe.

Your nearest doppelgänger is most likely to be much closer than these numbers suggest, given the processes of planet formation and biological evolution that tip the odds in your favor. Astronomers suspect that our Hubble volume has at least 10^100 habitable planets; some might well look like Earth.

The Level I multiverse framework is used routinely to evaluate theories in modern cosmology, although this procedure is rarely spelled out explicitly. For instance, consider how cosmologists used the microwave background to rule out a finite spherical geometry. Hot and cold spots in microwave background maps have a characteristic size that depends on the curvature of space, and the observed spots appear too small to be consistent with a spherical shape. But it is important to be statistically rigorous. The average spot size varies randomly from one Hubble volume to another, so it is possible that our universe is fooling us—it could be spherical but happen to have abnormally small spots. When cosmologists say they have ruled out the spherical model with 99.9 percent confidence, they really mean that if this model were true, fewer than one in 1,000 Hubble volumes would show spots as small as those we observe.
The lesson is that the multiverse theory can be tested and falsified even though we cannot see the other universes. The key is to predict what the ensemble of parallel universes is and to specify a probability distribution, or what mathematicians call a “measure,” over that ensemble. Our universe should emerge as one of the most probable. If not—if, according to the multiverse theory, we live in an improbable universe—then the theory is in trouble. I will discuss later, this measure problem can become quite challenging.

**Level II: Other Postinflation Bubbles**

If the Level I Multiverse was hard to stomach, try imagining an infinite set of distinct Level I multiverses, some perhaps with different spacetime dimensionality and different physical constants. Those other multiverses—which constitute a Level II multiverse—are predicted by the currently popular theory of chaotic eternal inflation.

Inflation is an extension of the big bang theory and ties up many of the loose ends of that theory, such as why the universe is so big, so uniform and so flat. A rapid stretching of space long ago can explain all these and other features in one fell swoop [see “The Inflationary Universe,” by Alan H. Guth and Paul J. Steinhardt; *Scientific American*, May 1994; and “The Self-Generating Multiverse,” by Andrei Linde, November 1994]. Such stretching is predicted by a wide class of theories of elementary particles, and all available evidence bears it out.

The phrase “chaotic eternal” refers to what happens on the very largest scales. Space as a whole is stretching and will continue doing so forever, but some regions of space stop stretching and form distinct bubbles, like gas pockets in a loaf of rising bread. Infinitely many such bubbles emerge. Each is an embryonic Level I multiverse: infinite in size and filled with matter deposited by the energy field that drove inflation.

Those bubbles are more than infinitely far away from Earth, in the sense that you would never get there even if you traveled at the speed of light forever. The reason is that the space between our bubble and its neighbors is expanding faster than you could travel through it. Your descendants will never see their doppelgängers elsewhere in Level II. For the same reason, if cosmic expansion is accelerating, as observations now suggest, they might not see their alter egos even in Level I.

The Level II multiverse is far more diverse than the Level I multiverse. The bubbles vary not only in their initial conditions but also in seemingly immutable aspects of nature. The prevailing view in physics today is that the dimensionality of spacetime, the qualities of elementary particles and many of the so-called physical constants are not built into physical laws but are the outcome of processes known as symmetry breaking. For instance, theorists think that the space in our universe once had nine dimensions, all on an equal footing. Early in cosmic history, three of them partook in the cosmic expansion and became the dimensions we now observe. The other six are now unobservable, either because they have stayed microscopic with a doughnutlike topology or because all matter is confined to a three-dimensional surface (a membrane, or simply “brane”) in the nine-dimensional space.

Thus, the original symmetry among the dimensions broke. The quantum fluctuations that drive chaotic inflation could cause different symmetry breaking in different bubbles. Some might become producing Inflationary Universes, as by Andrei Linde, November 1994). Such stretching is predicted by a wide class of theories of elementary particles, and all available evidence bears it out. The phrase “chaotic eternal” refers to what happens on the very largest scales. Space as a whole is stretching and will continue doing so forever, but some regions of space stop stretching and form distinct bubbles, like gas pockets in a loaf of rising bread. Infinitely many such bubbles emerge. Each is an embryonic Level I multiverse: infinite in size and filled with matter deposited by the energy field that drove inflation.

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Another way to produce a Level II multiverse might be through a cycle of birth and destruction of universes. In a scientific context, this idea was introduced by physicist Richard C. Tolman in the 1930s and recently elaborated on by Paul J. Steinhardt of Princeton University and Neil Turok of the University of Cambridge. The Steinhardt and Turok proposal and related models involve a second three-dimensional brane that is quite literally parallel to ours, merely offset in a higher dimension [see “Been There, Done That,” by George Musser; News Scan, *Scientific American*, March 2002]. This parallel universe is not

**Evidence**

Cosmologists infer the presence of Level II parallel universes by scrutinizing the properties of our universe. These properties, including the strength of the forces of nature (right) and the number of observable space and time dimensions (for right), were established by random processes during the birth of our universe. Yet they have exactly the values that sustain life. That suggests the existence of other universes with other values.
a wild stroke of luck. Stellar masses run from 10^30 to 10^32 kilograms. At first glance, this apparent coincidence of the habitable and observed mass values appears to be a lucky fluke. But the coincidence is 2.0 x 10^{-30}. What a coincidence, you say. After a moment of reflection, however, you conclude that this is not so surprising after all. For even if you knew nothing about hotels, you could infer the existence of other hotel rooms to explain the coincidence. 

As a more pertinent example, consider the mass of the sun. The mass of a star determines its luminosity, and using basic physics, one can compute that such a star as the sun is possible only if the sun's mass falls into the narrow range between 1.6 x 10^30 and 2.4 x 10^30 kilograms. Otherwise, the sun's climate would be colder than that of present-day Mars or hotter than that of present-day Venus. The measured solar mass is 2.0 x 10^30 kilograms. At first glance, this apparent coincidence of the habitable and observed mass values appears to be a wild stroke of luck. Stellar masses run from 10^30 to 10^32 kilograms, so if the sun acquired its mass at random, it had only a small chance of falling into the habitable range. But just as in the hotel example, one can explain this apparent coincidence by postulating an ensemble (in this case, a number of planetary systems) and a selection effect (the fact that we must find ourselves living on a habitable planet). Such observer-related selection effects are referred to as “anthropic,” and although the “A-word” is notorious for triggering controversy, physicists broadly agree that these selection effects cannot be neglected when testing fundamental theories.

What applies to hotel rooms and planetary systems applies to parallel universes. Most, if not all, of the attributes set by symmetry breaking appear to be fine-tuned. Changing their values even a little bit could blow itself apart before galaxies could form. Although the degree of fine-tuning is still debated, these examples suggest the existence of parallel universes with other values of the physical constants (see “Exploring Our Universe and Others,” by Martin Rees; Scientific American, December 1999). The Level II multiverse theory predicts that physicists will never be able to determine the values of these constants from first principles. They will merely compute probability distributions for what they should expect to find, taking selection effects into account. The result should be as generic as is consistent with our existence.

**Level III: Quantum Many Worlds**

The Level I and Level II multiverses involve parallel worlds that are far away, beyond the domain even of astronomers. But the next level of multiverse is right around you. It arises from the famous, and famously controversial, many-worlds interpretation of quantum mechanics—the idea that random quantum processes cause the universe to branch into multiple copies, one for each possible outcome.

In the early 20th century the theory of quantum mechanics revolutionized physics by explaining the atomic realm, which does not abide by the classical rules of Newtonian mechanics. Despite the obvious successes of the theory, a heated debate rages about what it really means. The theory specifies the state of the universe not in classical terms, such as the positions and velocities of all particles, but in terms of a mathematical object called a wave function. According to the Schrödinger equation, this state evolves over time in a fashion that mathematicians term “unitary,” meaning that the wave function rotates in an abstract infinite-dimensional space called Hilbert space. Although quantum mechanics is often described as inherently random and uncertain, the wave function evolves in a deterministic way. There is nothing random or uncertain about it.

The sticky part is how to connect this wave function with what we observe. Many legitimate wave functions correspond to counterintuitive situations, such as a cat being dead and alive at the same time in a so-called superposition. In the 1920s physicists explained away this weirdness by postulating that the wave function “collapsed” into some definite classical outcome whenever someone made an observation. This add-on had the virtue of explaining observations, but it turned an elegant, unitary theory into a kludgy, nonunitary one. The intrinsic randomness comcribed ascribed to quantum mechanics is the result of this postulate.

Over the years many physicists have abandoned this view in favor of one developed in 1957 by Princeton graduate student Hugh Everett III. He showed that the collapse postulate is unnecessary. Unadulterated quantum theory does not, in fact, pose any contradictions. Although it predicts that one classical reality gradually splits into superpositions of many such realities, observers subjectively experience this splitting merely as a slight randomness, with probabilities in exact agreement with those from the old collapse postulate. This superposition of classical worlds is the Level III multiverse.

Everett’s many-worlds interpretation has been boggling minds inside and outside physics for more than four decades. But the theory becomes easier to grasp when one distinguishes the world could be (within the scope of quantum mechanics) corresponds to a different universe. The parallel universes make their presence felt in laboratory experiments, such as wave interference and quantum computation.
between two ways of viewing a physical theory: the outside view of a physicist studying its mathematical equations, like a bird surveying a landscape from high above it; and the inside view of an observer living in the world described by the equations, like a frog living in the landscape surveyed by the bird. From the bird perspective, the Level I multiverse is simple. There is only one wave function. It evolves smoothly and deterministically over time without any kind of splitting or parallelism. The abstract quantum world described by this evolving wave function contains within it a vast number of parallel classical story lines, continuously splitting and merging, as well as a number of quantum phenomena that lack a classical description. From their frog perspective, observers perceive only a tiny fraction of this full reality. They can view their own Level I universe, but a process called decoherence—which mimics wave function collapse while preserving unitarity—prevents them from seeing Level III parallel copies of themselves.

Whenever observers are asked a question, make a snap decision and give an answer, quantum effects in their brains lead to a superposition of outcomes, such as “Continue reading the article” and “Put down the article.” From the bird perspective, the act of making a decision causes a person to split into multiple copies: one who keeps on reading and one who doesn’t. From their frog perspective, however, each of these alter egos is unaware of the others and notices the branching merely as a slight randomness: a certain probability of continuing to read or not. As strange as this may sound, the exact same situation occurs even in the Level I multiverse. You have evidently decided to keep on reading this article, but one of your alter egos in a distant galaxy put down the magazine after the first paragraph. The only difference between Level I and Level III is where your doppelgängers reside. In Level I they live elsewhere in good old three-dimensional space. In Level III they live on another quantum branch in infinite-dimensional Hilbert space.

The existence of Level III depends on one crucial assumption: that the time evolution of the wave function is unitary. So far experimenters have encountered no departures from unitarity. In the past few decades they have confirmed unitarity for ever larger particles, from the 60-hack-atom molecule and kilometer-long optical fibers. On the theoretical side, the case for unitarity has been bolstered by the discovery of decoherence [see “100 Years of Quantum Mysteries,” by Max Tegmark and John Archibald Wheeler; Scientific American, February 2001]. Some theorists who work on quantum gravity have questioned unitarity; one concern is that evaporating black holes might destroy information, which would be a nonunitary process. But a recent breakthrough in string theory known as AdS/CFT correspondence suggests that even quantum gravity is unitary. Lattice gauge theories do not describe quantum gravity but merely transmit it elsewhere. [Editors’ note: An upcoming article will discuss this correspondence in greater detail.]

If physics is unitary, then the usual standard picture of how quantum fluctuations operated early in the Big Bang must change. These fluctuations did not generate initial conditions at random. Rather they generated a quantum superposition of all possible initial conditions, which coexisted simultaneously. De-coherence then caused these initial conditions to behave classically in separate quantum branches. Here is the crucial point: the distribution of outcomes on different quantum branches in a given Hubble volume (Level III) is identical to the distribution of outcomes in different Hubble volumes within a single quantum branch (Level I). This property of the quantum fluctuations is known in statistical mechanics as ergodicity. The same reasoning applies to Level II. The process of symmetry breaking did not produce a unique outcome but rather a superposition of all outcomes, which rapidly went their separate ways. So if physical constants, spacetime dimensionality and so on can vary among parallel quantum branches at Level III, then they will also vary among parallel universes at Level II.

In other words, the Level III multiverse adds nothing new beyond Level I and Level II, just more indistinguishability copies of the same universes—the same old story lines playing out again and again in other quantum branches. The passionate debate about Everett’s theory therefore seems to be ending in a grand anticlimax, with the discovery of less controversial multiverses (Levels I and II) that are equally large.

Needless to say, the implications are profound, and physicists tend to be Platonists, suspecting that mathematics describes the universe so well because the universe is inherently mathematical. Then all of physics is ultimately a mathematics problem: a mathematician with unlimited intelligence and resources could in principle compute the frog perspective of an observer within it. Unfortunately, it is not an easy task to compute what fraction of the infinitely many observers perceive what. The answer depends on the order in which you count them. By analogy, the fraction of the integers that are even is 50 percent if you order them numerically [1, 2, 3, 4, ...] but approaches 100 percent if you sort them by digit by digit, the way your word processor would [1, 10, 1, 100, 1, 1000, ...]. When observers reside in disconnected universes, there is no obviously natural way in which to order them. Instead one must sample from the different universes with some statistical weights referred to by mathematicians as a “measure.” This problem crops up in a mild and treatable manner at Level I, but becomes severe at Level II, has caused much debate at Level III, and is horrendous at Level IV. At Level II, for instance, Alexander Vilenkin of Tufts University and others have published predictions for the probability distributions of various cosmological singularities. They have argued that there are two different parallel universes that have inflated by different amounts should be given statistical weights proportional to their volume. On the other hand, any mathematician will tell you that 2

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= ∞, so there is no objective sense in which an infinite universe that has expanded by a factor of two has gotten larger. Moreover, a finite universe with the topology of a torus is equivalent to a perfectly periodic universe with infinite volume, both from the mathematical bird perspective and from the frog perspective of an observer within it. So why should its infinitely smaller volume give it zero statistical weight? After all, even in the Level I multiverse, Hubble volume starts repeating [albeit in a random order, not periodically] after about 10 to the 1018 meters. If you think that is bad, consider the problem of assigning statistical weights to different mathematical structures at Level IV: The fact that our universe seems relatively simple has led many people to suggest that the correct measure somehow involves complexity. —M.T.
In four-dimensional spacetime—the bird perspective—these particle trajectories resemble a tangle of spaghetti. If the frog sees a particle moving with constant velocity, the bird sees a straight strand of uncooked spaghetti. If the frog sees a pair of orbiting particles, the bird sees two spaghetti strands intertwined like a double helix. To the frog, the world is described twined like a double helix. To the bird, the world is described orbiting particles, the bird sees two spaghetti strands inter—

In the coming decade, dramatically improved cosmological measurements of the microwave background and the large-scale matter distribution will support or refute Level I by further pinning down the curvature and topology of space. These measurements will also probe Level II by testing the theory of chaotic eternal inflation. Progress in both astrophysics and high-energy physics should also clarify the extent to which physical constants are fine-tuned, thereby weakening or strengthening the case for Level II.

If current efforts to build quantum computers succeed, they will provide further evidence for Level III, as they would, in essence, be exploiting the parallelism of the Level III multiverse for parallel computation. Experimenters are also looking for evidence of unitarity violation, which would rule out Level III. Finally, success or failure in the grand challenge of modern physics—unifying general relativity and quantum field theory—will sway opinions on Level IV. Either we will find a mathematical structure that exactly matches our universe, or we will bump up against a limit to the unreasonable effectiveness of mathematics and have to abandon that level.

So should you believe in parallel universes? The principal arguments against them are that they are wasteful and that they are weird. The first argument—that multiverse theories are vulnerable to Occam’s razor because they postulate the existence of other worlds that we can never observe. Why should nature be so wasteful and indulge in such opulence an infinity of different worlds? Yet this argument can be turned around to argue for a multiverse. What precisely would nature be wasting? Certainly not space, mass or atoms—the uncontroversial Level I multiverse already contains an infinite amount of all three, so who cares if nature wastes some more? The real issue here is the apparent reduction in simplicity. A skeptic worries about all the information necessary to specify all those unseen worlds. But an entire ensemble is often much simpler than one of its members. This principle can be stated more formally using the notion of algorithmic information content. The algorithmic information content in a number is, roughly speaking, the length of the shortest computer program that will produce that number as output. For example, consider the set of all integers.

Which is simpler, the whole set or just one number? Naively, you might think that a single number is simpler, but the entire set can be generated by quite a trivial computer program, thereby losing the simplicity and symmetry that were inherent in the totality of all the elements taken together.

In this sense, the higher-level multiverses are simpler. Going from our universe to the Level I multiverse eliminates the need to specify initial conditions, upgrading to Level II eliminates the need to specify physical constants, and the Level IV multiverse eliminates the need to specify anything at all. The opulence of complexity is all in the subjective perceptions of observers—the frog perspective. From the bird perspective, the multiverse could hardly be any simpler.

The complaint about weirdness is aesthetic rather than scientific, and it really makes sense only in the Aristotelian worldview. Why, then, do we expect it? When we ask a profound question about the nature of reality, do we not expect an answer that sounds strange? Evolution provided us with intuition for the everyday physics that had survival value for our distant ancestors, so whenever we venture beyond the everyday world, we should expect it to seem bizarre.

A common feature of all four multiverse levels is that the simplest and arguably most elegant theory involves parallel universes by default. To deny the existence of those universes, one needs to complicate the theory by adding experimentally unsupported processes and ad hoc postulates: finite space, wave function collapse and ontological asymmetry. Our judgment therefore comes down to which we find more wasteful and inelegant: many worlds or many words. Perhaps we will gradually get used to the weird ways of our cosmos and find its strangeness to be part of its charm.

encouraging feature of mathematical structures is that the symmetry and invariance properties that are responsible for the simplicity and orderliness of our universe tend to be generic, not just local in the exception. Mathematical structures do not reside in the same space but exist outside of space and time; they are almost impossible to visualize; the best one can do is to think of them abstractly, as static sculptures that represent the mathematical structure of the physical laws that govern them. For example, consider a simple universe: Earth, moon and sun, obeying Newton’s laws. To an objective observer, this universe looks like a circular ring (Earth’s orbit smeared out in time) wrapped in a braid (the moon’s orbit around Earth). Other shapes embody other laws of physics (a, b, c, d). This paradigm solves various problems concerning the foundations of physics.

THE ULTIMATE TYPE of parallel universe opens up the full realm of possibility. Universes can differ not just in location, cosmological properties or quantum state but also in the laws of physics. Existing outside of space and time, they are almost impossible to visualize; the best one can do is to think of them abstractly, as static sculptures that represent the mathematical structure of the physical laws that