Spectral diagnostics for IXO

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1 Introduction

IXO with its high throughput and spectral resolution will allow X-ray spectroscopy to become a standard working horse for astrophysics. The grating spectrometers on Chandra and XMM-Newton have opened the field, but due to their limited effective area only for the brightest sources and due to the way that gratings work only for point sources or very compact extended sources. With IXO, the effective area increases by 3 orders of magnitude with roughly similar spectral resolution. Due to the use of imaging TES arrays, also extended sources can be studied without complexities.

The current design of IXO is based on both a calorimeter array as well as a grating spectrometer. In this document we compare the performance of both types of instruments for different astrophysical situations in a generic way.

2 General spectral line diagnostics: limiting sensitivities

2.1 Line flux measurements

2.1.1 Basic definitions

Consider a spectrum with a well isolated spectral line at energy $E$ with flux $F_\ell$ (phot m$^{-2}$ s$^{-1}$) superimposed on a continuum with flux $F_c$ (phot m$^{-2}$ s$^{-1}$ keV$^{-1}$). The line can be either an emission line or an absorption line. The equivalent width $W$ of the line (in keV) is then given by

$$ W \equiv \frac{F_\ell}{F_c}. \quad (1) $$

Suppose that this line is observed with an exposure time $t$ (in s), with an effective area $A$ (m$^2$) and instrumental resolution $\Delta E$ (in keV). The number of line photons $N_\ell$ is then given by

$$ N_\ell = F_\ell A t \quad (2) $$

and the number of continuum photons $N_c$ within the resolution element $\Delta E$ is given by

$$ N_c = F_c A t \Delta E. \quad (3) $$

In addition there may be an instrumental background $F_b$ that formally can also be expressed in phot m$^{-2}$ s$^{-1}$ keV$^{-1}$ and that gives $N_b$ background counts within a resolution element:

$$ N_b = F_b A t \Delta E. \quad (4) $$

The total number of counts $N$ within the bin containing the line is then given by

$$ N = N_b + N_c + N_\ell. \quad (5) $$
2.1.2 Minimum number of photons needed

Using Poissonian statistics, \( N \) can be determined with an accuracy (r.m.s. error) of \( \sqrt{N} \). Therefore, to detect a line with a minimum signal to noise ratio \( S = N_\ell/\sqrt{N} \), one must have

\[
N_\ell > S \sqrt{N_0 + N_c + N_\ell}.
\]  
(6)

By squaring (6), dividing it by \( N_\ell \) and substituting (2)–(4) and (1) this equation can be rewritten as:

\[
N_\ell > S^2 \left[ 1 + \frac{\Delta E}{W} \left( 1 + \frac{F_b}{F_c} \right) \right].
\]  
(7)

This equation (7) immediately tells that high spectral resolution (small \( \Delta E \)) and low background (small \( F_b \)) are favourable conditions for detecting lines. We can easily define an effective spectral resolution \( \tilde{\Delta}E \) to account for the background as

\[
\tilde{\Delta}E \equiv \Delta E \left( 1 + \frac{F_b}{F_c} \right).
\]  
(8)

We distinguish now the special cases of strong and weak lines. Strong lines are defined here as having \( W \gg \tilde{\Delta}E \) and weak lines as having \( W \ll \tilde{\Delta}E \). Inserting these limiting cases into (7) one gets

\[
\begin{align*}
\text{strong lines, } W \gg \tilde{\Delta}E: & \quad N_\ell > S^2 \quad \text{(9)} \\
\text{weak lines, } W \ll \tilde{\Delta}E: & \quad N_\ell > S^2 \left( \frac{\Delta E}{W} \right). \quad \text{(10)}
\end{align*}
\]

Taking \( S = 5 \), we need for a strong line at least 25 counts to get a 5\( \sigma \) detection, and for weaker lines even more.

The above estimates hold both for emission lines and absorption lines, provided one takes formally a negative line flux \( F_\ell \). For estimating the sensitivity for weak absorption lines, in general the underlying continuum spectrum is the most important factor. By using the relation \( N_c = N_\ell (\Delta E/W) \), (10) can be rewritten as

\[
\text{weak lines, } W \ll \tilde{\Delta}E: \quad N_c > S^2 \left( \frac{\Delta E}{W} \right)^2 \left( 1 + \frac{F_b}{F_c} \right) \quad \text{(11)}
\]

For example (ignoring here the background), for a 5\( \sigma \) detection of an absorption line with an equivalent width 10% of the spectral resolution element (hence with a measured depression of order 10% with respect to the continuum), one needs at least 2500 counts per resolution element, or a nominal signal to noise in the continuum of 50. This shows that weak line detection is not impossible, but is facilitated by either high spectral resolution or high effective area.

2.1.3 Systematic uncertainties

By arbitrarily enhancing the effective area or the integration time for an instrument, it is not possible to detect the weakest absorption or emission lines. The ultimate sensitivity is limited by the systematic bin to bin variations of the continuum spectrum.

There can be various reasons for such bin to bin variations. First, there can be instrumental reasons. For instance, in the case of a CCD detector behind a grating, like the RGS detector of XMM-Newton, unknown small-scale spatial variations of the CCD quantum efficiency or filter transmission can cause pixel to pixel variations. But perhaps even more important, small effects from unrecognised weak "warm" pixels could play a role. But also for TES detectors tiny unaccounted, energy-dependent factors may be present, in particular near instrumental edges.

Secondly, there can be astrophysical reasons for bin to bin variations of the continuum. For instance, for ISM absorption studies one might assume that the background source used may have a perfect power law,
but the source might also have weak intrinsic spectral features. For a grating spectrometer, the presence of another weak emission line source along the dispersion axis may contaminate the continuum of the primary source. Often one can recognise this or account for it, for instance by choosing a different roll angle. But even for imaging TES detectors, the source extraction region may contain a weak foreground or background source. Typical line-rich sources may have peak fluxes in their strongest lines that are ten times brighter than the broad-band average flux. Hence, when studying a given target, a line-emitting background source with only $\sim 0.1\%$ of the flux of the main target may give contaminating line fluxes of $\sim 1\%$ of the continuum flux of the target.

Finally, there are limitations to the completeness and accuracy of spectral codes. Not all weak spectral features are taken into account. Whatever the reason for the systematic uncertainties is (instrumental, background or astrophysical), they limit the ultimate line sensitivity. In most practical cases, going beyond a systematic relative continuum uncertainty $u < 0.01$ seems difficult.

It is then easy to show that for weak spectral lines the minimum detectable equivalent width $W_{\text{min}}$ is given by

$$W_{\text{min}} = S u \Delta E,$$  \hspace{1cm} (12)

with $S$ playing here the role of the required significance. For typical numbers $S = 5$ and $u = 0.01$, no lines weaker than $\sim 5\%$ of the instrumental resolution $\Delta E$ can be detected.

### 2.1.4 Figure of merit of an instrument for line detection

For a comparison of different instruments with respect to detecting spectral lines, the figure of merit is simply proportional to the signal to noise ratio $S$. Using the formalism derived above, we can write $S$ as

$$S = \frac{\sqrt{F_A t}}{\sqrt{1 + \frac{\Delta E}{W}}}.$$  \hspace{1cm} (13)

We now define the figure of merit $M_\ell$ for line detection as

$$M_\ell \equiv S / \sqrt{F_\ell t}.$$  \hspace{1cm} (14)

An instrument with a figure of merit $n \times$ higher than another instrument, will get an $n \times$ higher detection significance for the same line within the same exposure time. We now consider the two limiting cases for strong and weak lines. Using (13) and (14) we obtain the result

- strong lines, $W \gg \Delta E$: \hspace{1cm} $M_\ell = \sqrt{A},$  \hspace{1cm} (15)
- weak lines, $W \ll \Delta E$: \hspace{1cm} $M_\ell = \sqrt{A / \Delta E}.$  \hspace{1cm} (16)

Here for weak lines we have eliminated the irrelevant factor $\sqrt{W}$ and the dependence on the background $F_b$, which is justified for sufficiently strong continuum sources.

For grating spectrometers where more than one spectral order is available, one should adjust the above equations as follows. The figure of merit is proportional to the signal to noise ratio $S$. Using the addition rules for errors, the total value of $S$ determined from combining a number of spectral orders is given by

$$S = \sqrt{\sum_{i=1}^{n} S_i^2}.$$  \hspace{1cm} (17)

For gratings with both positive and negative spectral orders, the summation must obviously be extended over both signs. Thus, a generalisation of (15) and (16) is given by

- detection strong lines, $W \gg \Delta E$: \hspace{1cm} $M_\ell = \sqrt{\sum_{i=1}^{n} A_i},$  \hspace{1cm} (18)
detection weak lines, $W \ll \tilde{\Delta}E$: 

$$M_{\ell} = \sqrt{\sum_{i=1}^{n} \frac{A_i}{\Delta E_i}}.$$  \hspace{2cm} (19)

In the above equations there is one caveat, namely that in some cases a line may be weak in first spectral order, but strong in a higher spectral order, because of the increased spectral resolution in the higher spectral orders.

For grating spectrometers, in general one can approximate $\Delta E_i = \Delta E_1/i$, where $i$ is the spectral order, and therefore the higher spectral orders may contribute significantly to (19).

2.2 Line centroids

2.2.1 Basic estimates

Detection of a line is the first step of analysis. Once the line has been detected with sufficient significance, it can be used as a diagnostic tool. Here we focus on the role of relatively well isolated spectral lines as diagnostics for measuring velocity fields.

For an instrument with a Gaussian redistribution function with r.m.s. width $\sigma_0$, line centroids in the strong line approximation can be determined with an accuracy of $\sigma_0/\sqrt{N_\ell}$. If there is a strong continuum, this becomes more complicated. As a simple approximation, we can replace this expression however by $\sigma_0\sqrt{N_\ell+N_c}/N_\ell$, or alternatively using the definition of the S/N ratio $S$ as $\sigma_0/S$. Taking now $\Delta E = 2.35\sigma_0$ and transforming into a velocity scale, we find that the line centroid can be measured with an accuracy $\Delta v$ (km s$^{-1}$) of

$$\Delta v = \frac{c\Delta E}{2.35ES}. \hspace{2cm} (20)$$

2.2.2 Systematic uncertainties

For measuring velocity fields, the dominant source of systematic uncertainty is the absolute accuracy of the energy scale. Let us consider here an example. For a not too strong oxygen line with $\Delta E = 2$ eV, $S = 5$ and $E = 0.5$ keV we get $\Delta v = 102$ km/s, and for an iron line $\Delta E = 5$ eV, $S = 5$ and $E = 6.4$ keV we have $\Delta v = 20$ km/s. But to achieve this accuracy, the energy scale must be known with an accuracy of 1 in 3 000 and 1 in 15 000, respectively, or 0.17 and 0.43 eV. For stronger lines, the systematic limit will be reached more rapidly.

2.2.3 Figure of merit of an instrument for velocity fields

From our discussion in Sect. 2.2.1 it follows that a natural choice to define a figure of merit $M_\nu$ for velocity diagnostics would be to take a quantity that is proportional to $c/\Delta v$; the higher this quantity is, the better (more accurate) is the detector. Thus, from (20) we find that $M_\nu$ should be proportional to $ES/\Delta E$. From this we find using (13):

$$\text{strong lines, } W \gg \tilde{\Delta}E: \quad M_\nu = \frac{E\sqrt{A}}{\Delta E},$$ \hspace{2cm} (21)

$$\text{weak lines, } W \ll \tilde{\Delta}E: \quad M_\nu = \frac{E\sqrt{A}}{(\Delta E)^{1.5}}. \hspace{2cm} (22)$$

For gratings we need to combine again the different spectral orders. How to do this can be seen as follows. The weighted mean velocity $\bar{v}$ is given by $\bar{v} \equiv \sum w_i v_i / \sum w_i$ with $w_i = \Delta v_i^{-2}$. Calculating
the variance of this expression yields the error on the mean velocity as $\Delta \bar{v} = 1/\sqrt{\sum w_i}$. Substituting $w_i = \Delta v_i^{-2}$ and using (20) we find for the figure of merit in general

$$M_v = \sqrt{\sum (E_i S_i/\Delta E_i)^2}. \tag{23}$$

Thus, a generalisation of (21) and (22) is given by

- **velocity strong lines,** $W \gg \tilde{\Delta} E$:
  $$M_v = \frac{\sum_{i=1}^{n} E_i^2 A_i}{(\Delta E_i)^2}. \tag{24}$$

- **velocity weak lines,** $W \ll \tilde{\Delta} E$:
  $$M_v = \frac{\sum_{i=1}^{n} E_i^2 A_i}{(\Delta E_i)^3}. \tag{25}$$

### 2.3 Line broadening

Measuring line broadening is important for measuring turbulence or thermal line broadening. When the line broadening is larger than the resolution of the instrument, measuring the line width is trivial. However, when the line width is small compared to the resolution, the limiting broadening is determined by the statistics of the spectrum and the resolution.

We start here with some basic statistics. For $n$ drawings from a normal (Gaussian) distribution with arbitrary mean and variance $\sigma^2$, and in the limit of large $n$, the uncertainty $\Delta \sigma$ in the estimated r.m.s. width $\sigma$ is given by

$$\Delta \sigma = \frac{\sigma}{\sqrt{2n}}. \tag{26}$$

This can be seen as follows. If $s^2$ is the best estimate of $\sigma^2$ based on the data ($s^2 = \sum (x_i - \bar{x})^2/(n-1)$), then the quantity $ns^2/\sigma^2$ has a $\chi^2$ distribution with $n-1$ degrees of freedom. Since this $\chi^2$ distribution has mean $n-1$ and variance $2(n-1)$, in the limit of large $n$, $s^2$ has a normal distribution with mean $\sigma^2$ and variance $2\sigma^2/n$. Hence the r.m.s. uncertainty on $s^2$ is $(2/n)^{0.5}\sigma^2$. From the rules of error propagation we know that $\Delta(s^2) = 2s\Delta(s)$. Using this, (26) is simply obtained.

For a spectrum with zero background, (26) can be used directly to estimate the uncertainty in the width of the distribution. For non-zero background, it is easily generalised to

$$\Delta \sigma = \frac{\sigma}{\sqrt{2S}}, \tag{27}$$

where we used again the signal to noise ratio $S$ at the position of the line.

For an instrument with Gaussian redistribution with r.m.s. width $\sigma_0$, and spectral lines with intrinsic Gaussian broadening $\sigma_s$, we should use here

$$\sigma^2 = \sigma_0^2 + \sigma_s^2. \tag{28}$$

This implies that $\sigma d\sigma = \sigma_s d\sigma_s$ and combining this with (27) and (28) it follows that

$$\Delta \sigma_s = \frac{\sigma_0^2 + \sigma_s^2}{\sqrt{2\sigma_s S}}. \tag{29}$$

#### 2.3.1 Systematic uncertainties

Also in the case of measuring intrinsic line width, the ultimate sensitivity is limited by instrumental effects. In the present case, it is simply the uncertainty in the r.m.s. width $\sigma_0$ relative to the energy $E$ of the line. Calibrating the line spread function is often one of the last and most difficult tasks that is performed for any instrument.
2.3.2 Figure of merit of an instrument for velocity broadening

We now can determine the figure of merit for measuring the velocity broadening of narrow spectral lines. For narrow spectral lines, we have \( \sigma_s \ll \sigma_0 \) and hence (29) reduces to

\[
\Delta \sigma_v = \frac{c^2 (\Delta E)^2}{7.84 \sigma_v E^2 S},
\]

where we have made the transition from energy units \( \sigma_s \) to velocity units \( \sigma_v \) using \( \sigma_v / c = \sigma_s / E \) and we also used \( \Delta E = 2.35 \sigma_0 \).

The figure of merit \( M_\sigma \) in this case is obviously a quantity proportional to \( 1/\sigma_v \). Therefore we have

\[
M_\sigma \sim \frac{E^2 S}{(\Delta E)^2}.
\]

Substituting the limiting cases for \( S \) for strong and weak lines, we have

\[
\text{strong lines, } W \gg \tilde{\Delta} E: \quad M_\sigma = \frac{E^2 \sqrt{A}}{(\Delta E)^2},
\]

\[
\text{weak lines, } W \ll \tilde{\Delta} E: \quad M_\sigma = \frac{E^2 \sqrt{A}}{(\Delta E)^{2.5}}.
\]

For gratings we need to combine again the different spectral orders. Similar to what we found for the figures of merit for line detection and for measuring the line centroids, the total figure of merit is obtained by adding quadratically the \( M'_s \) of the individual spectral orders and taking the square root of this sum:

\[
\text{broadening strong lines, } W \gg \tilde{\Delta} E: \quad M_\sigma = \left( \sum_{i=1}^{n} \frac{E_i^4 A_i}{(\Delta E_i)^4} \right)^{1/2},
\]

\[
\text{broadening weak lines, } W \ll \tilde{\Delta} E: \quad M_\sigma = \left( \sum_{i=1}^{n} \frac{E_i^5 A_i}{(\Delta E_i)^5} \right)^{1/2}.
\]

It is seen from these expression that a small improvement in spectral resolution has a huge impact on the capability to measure line broadening.

2.4 Disentangling line blends

Frequently examples occur where the structure of a line complex is important to know. For instance, is there one dynamical component with a slight amount of line broadening, or are there two narrow and blended velocity components? Such problems usually all boil down to the question of how close two competing spectral models are. It can be shown that the limiting factor in these cases is always given by the resolution of the instrument. The question is related to variants of the information theorem of Shannon (1949, Proc. IRE 37, 10): what is the minimum bin size for an observed spectrum without losing information on the underlying model?

We cannot go here into detail on the Shannon theorem. A rule of thumb that is often applied is that data should not be binned into bins smaller than \( 1/3 \) of the instrumental FWHM. The number \( 1/3 \) is not very precise; in fact, it depends on the shape of the instrumental redistribution function, on the number of counts in a line and on the number of resolution elements in the full spectrum. But it depends only
weakly on those parameters. For more discussion on these issues and a more robust estimate of the precise factor we refer to the SPEX user’s manual\(^1\).

However, we give here an example to show that in practice \(\Delta E\) is indeed the limiting parameter. Suppose we want to compare two spectral models. Model 1 has a normal distribution with mean 0 and variance \(1 + \epsilon^2\), with \(\epsilon \ll 1\). Model 2 consists of the sum of two spectral lines, both with a normal distribution with variance 1 but centres \(-\epsilon\) and \(+\epsilon\), respectively. The probability density functions for both cases are given by

\[
\begin{align*}
  f_1(x) &= \frac{1}{\sqrt{2\pi(1 + \epsilon^2)}} e^{-x^2/2(1 + \epsilon^2)}, \\
  f_2(x) &= \frac{1}{\sqrt{2\pi}} \left[ \frac{1}{2} e^{-(x-\epsilon)^2/2} + \frac{1}{2} e^{-(x+\epsilon)^2/2} \right].
\end{align*}
\]

Both distributions \(f_1(x)\) and \(f_2(x)\) have the same flux (1), mean (0) and variance \((1 + \epsilon^2)\). After some straightforward algebra it can be shown that the ratio

\[
\frac{f_2(x)}{f_1(x)} = \sqrt{1 + \epsilon^2 \cosh(\epsilon x)} e^{-\frac{\epsilon^2}{2} \left[ 1 + \frac{x^2}{1 + \epsilon^2} \right]}.
\]

Making a Taylor’s expansion of this expression up to 4th order in \(\epsilon\), it is easy to show that the relative difference \(\delta(x)\) between both distributions is given by

\[
\delta(x) = \frac{f_2(x)}{f_1(x)} - 1 = -\frac{\epsilon^4}{4} (1 - 2x^2 + \frac{1}{3}x^4).
\]

A comparison of both distributions is usually based on a quantity like

\[
\lambda = N \int_{-\infty}^{+\infty} f_1(x)\delta(x)^2 \, dx,
\]

where \(N\) is the total number of counts in the line (blend). Significant differences can be detected if \(\lambda\) is typically larger than unity. Therefore, given the \(\epsilon^4\) dependence of \(\delta\), one needs of order \(N = \epsilon^{-8}\) counts to be able to discriminate between \(f_1\) and \(f_2\). For \(\epsilon = 1/3\), this is already \(\sim 10^4\) counts.

In conclusion, the above shows that it is never possible to find a unique solution for the fine structure of a spectrum at scales a few times smaller than the resolution element \(\Delta E\).

Therefore, we may define the figure of merit for resolving line structure simply as

\[
\text{unresolved line structure: } M_u = \frac{E}{\Delta E}.
\]

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\(^1\)http://www.sron.nl/divisions/hea/spex/version2.0/release/manual.ps.gz
2.5 Summary of figures of merit

Here we summarise the figures of merit derived in the previous sections.

\[
M_{\ell} = \sqrt{\sum_{i=1}^{n} A_i},
\]

(42)

\[
detection\ strong\ lines,\ W \gg \tilde{\Delta}E:
\]

(43)

\[
detection\ weak\ lines,\ W \ll \tilde{\Delta}E:
\]

(44)

\[
velocity\ strong\ lines,\ W \gg \tilde{\Delta}E:
\]

(45)

\[
velocity\ weak\ lines,\ W \ll \tilde{\Delta}E:
\]

(46)

\[
broadening\ strong\ lines,\ W \gg \tilde{\Delta}E:
\]

(47)

\[
broadening\ weak\ lines,\ W \ll \tilde{\Delta}E:
\]

(48)

\[
unresolved\ line\ structure:\ M_u = \frac{E}{\Delta E}
\]

3 Comparing the spectral capabilities of the IXO TES array and grating spectrometer

We now make some simple estimates based upon the numbers as presented at the IXO meeting in September 2008 and data available at the IXO website. For the TES array (XRC) we used the effective area from the response matrix ixouncal05081030.rsp (status 17 November 2008), which has spectral resolution \(\Delta E = 2.5\) eV. For the gratings (XGS) we use \(A = 0.1\) m\(^2\) (option 1) or \(A = 0.1\) m\(^2\) (option 2), with \(R = E/\Delta E = 3000\). The effective areas are shown in Fig. 1.

For our estimates, we will choose \(E = 0.5\) keV, hence \(\Delta E = 0.17\) eV for the XGS and \(R = E/\Delta E = 200\) for the XRC. For comparison, we also give here the numbers for the RGS of XMM-Newton and the LETG and HETG of Chandra, all instrument also at 0.5 keV.

Inserting these numbers into the equations derived in the previous section then leads to the following figures of merit (we use areas in m\(^2\) and energies in keV):

For detecting weak lines, the XRC and XGS have comparable performance, but for anything that involves dynamics (velocities, broadening, line structure) the XGS is superior by 1 to 2 orders of magnitude.

It should be noted that the performance of the gratings for dynamics may be even better than shown above, as our estimates above are based on a single effective area and spectral resolution, while in practice a weighted averaging over the spectral orders should be done. In particular for the higher moments, the higher spectral orders are very important.
Table 1: Comparison of figures of merit for $E = 0.5$ keV. For Chandra LETG, the HRC-S detector is used. For XMM-Newton RGS, both RGS detectors are taken into account. For both Chandra and XMM-Newton, only the first spectral orders are taken into account. For the grating spectrometer of IXO (XGS), two options are worked out: option 1: 0.1 m$^2$ effective area; option 2: 0.3 m$^2$ effective area. Also the calorimeter array (XRC) is shown. The last column shows the ratio of the numbers for XGS (option 2) to XRC.

<table>
<thead>
<tr>
<th>Satellite: Instrument:</th>
<th>Chandra LETG</th>
<th>Chandra HETG</th>
<th>XMM RGS</th>
<th>IXO XGS$^1$</th>
<th>IXO XGS$^2$</th>
<th>IXO XRC</th>
<th>XGS$^2$/XRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (m$^2$)</td>
<td>0.0017</td>
<td>0.00026</td>
<td>0.0095</td>
<td>0.1</td>
<td>0.3</td>
<td>1.09</td>
<td>0.27</td>
</tr>
<tr>
<td>$R$</td>
<td>600</td>
<td>1100</td>
<td>380</td>
<td>3000</td>
<td>3000</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>$\Delta E$ (eV)</td>
<td>0.84</td>
<td>0.45</td>
<td>1.3</td>
<td>0.17</td>
<td>0.17</td>
<td>2.5</td>
<td>0.07</td>
</tr>
<tr>
<td>$\Delta \lambda$ (mÅ)</td>
<td>40</td>
<td>20</td>
<td>65</td>
<td>8.3</td>
<td>8.3</td>
<td>120</td>
<td>0.07</td>
</tr>
<tr>
<td>$W_{\text{min}}$ (eV)</td>
<td>0.042</td>
<td>0.023</td>
<td>0.066</td>
<td>0.008</td>
<td>0.008</td>
<td>0.125</td>
<td>0.07</td>
</tr>
<tr>
<td>$W_{\text{min}}$ (mÅ)</td>
<td>2.1</td>
<td>1.1</td>
<td>3.3</td>
<td>0.41</td>
<td>0.41</td>
<td>6.2</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Figures of merit:

- detection strong lines
- detection weak lines
- velocity strong lines
- velocity weak lines
- broadening strong lines
- broadening weak lines
- unresolved line structure
For the weakest lines, we have also seen that the minimum detectable equivalent width $W_{\text{min}}$ due to systematic uncertainties is given by (12). Demanding at least $5\sigma$ detections ($S = 5$) above this systematic limit, and adopting a systematic continuum uncertainty $u = 0.01$ we find the values listed in Table 1. Despite the fact that the XRC has 640 and 115 times more effective area compared to LETG and RGS, respectively, its minimum detectable equivalent width is 3 and 2 times higher than for these Chandra and XMM-Newton instruments. Only with the XGS a significantly smaller equivalent width can be reached.

The above was based on a single energy (0.5 keV). We have also calculated the results for all relevant energies, and show the results in Figs. 2–6.
Figure 2: Minimum detectable equivalent width $W_{\min}$ for strong (top) or weak (bottom) spectral lines for 1 % systematic uncertainties in the continuum.
Figure 3: Figure of merit for detecting strong (top) or weak (bottom) spectral lines.
Figure 4: Figure of merit for measuring velocities for strong (top) or weak (bottom) spectral lines
Figure 5: Figure of merit for measuring line broadening for strong (top) or weak (bottom) spectral lines.
Figure 6: Figure of merit for unraveling unresolved line structure