COMPLEXITY IN SPACE PLASMAS – A BRIEF REVIEW

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Abstract. Recent developments of the physics of complexity in space plasmas are briefly reviewed. The definition of dynamical complexity is provided. Concepts of probability distribution functions, wavelet transforms, intermittent turbulence, multifractal processes and extreme events are described. Future directions for this emerging field are discussed.

Keywords: dynamical complexity, multifractals, intermittent turbulence

1. Introduction

The phenomenon of criticality and complexity has had a long history in condensed matter physics. In parallel, the concepts of finite-dimensional chaos for nonequilibrium dissipative systems in terms of the ideas of strange attractors have received considerable attention by the researchers in nonlinear dynamics, since the ideas were first introduced by Lorenz (1963). Related ideas to dynamical systems far from equilibrium were popularized later by a seminal paper where the term "self-organized criticality" was first introduced (Bak *et al.*, 1987). On the other hand, applications of the above ideas to space plasmas were relatively recent. Probably the first contact of these ideas with space plasmas was contained in a series of papers addressing the low-dimensional chaos of the dynamics of the Earth's magnetosphere (Baker *et al.*, 1990; Shan *et al.*, 1991; Roberts *et al.*, 1991; Klimas *et al.*, 1992, Sharma *et al.*, 1993). These ideas were followed by a paper suggesting the possibility of interpreting such phenomenon from the point of view of forced and/or self-organized criticality using the method of the dynamic renormalization group (Chang, 1992). The idea was motivated by the low-dimensional chaos papers as well as the interpretation of the observations of sporadic and localized current disruptions in the Earth magnetotail (Lui, 1991). At approximately the same time, Lu and Hamilton (1991) also used the ideas of self-organized criticality to explain the curious scale-independent power-law behavior of the solar flare occurrence rate on the flare sizes. Almost simultaneously, Burlaga (1991) began to interpret the turbulent behavior of the solar wind from the point of view of multifractals. These ideas

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were followed by Consolini *et al.* (1996) and Consolini (1997) in their applications to the observed fractal time series of the AE indices.

Due to the limitation of space, this brief review of the concepts of complexity in space plasmas will be phrased in terms of answers to five frequently asked questions (FAQ) listed below.

- What do we mean by “dynamical complexity”?
- How is dynamical complexity related to “intermittent turbulence”?
- What is “magnetic reconfiguration”?
- What are “multifractals”?
- What are “extreme events”?

2. Dynamical Complexity

We begin our discussions with the definition of “dynamical complexity.” “Dynamical complexity is a phenomenon exhibited by a nonlinearly interacting dynamical system within which multitudes of different sizes of large scale “coherent structures” are formed, resulting in a global nonlinear stochastic behavior for the dynamical system, which is vastly different from that could be surmised from the original dynamical equations.”

Therefore, the ability to form multitudes of varieties of large scale coherent structures of different sizes in a nonlinear dynamical system is the prerequisite for the onset of the phenomenon of dynamical complexity. In space, by large scale we mean sizes much larger than the original particle (ions, electrons, and neutrals) sizes of the plasma medium. For plasmas, it is known that because of the intrinsic nonlinearity in the original dynamical equations, many kinds of coherent structures can be formed. Generally, such coherent structures may take on the shapes of convective forms, nonlinear solitary structures, pseudo-equilibrium configurations, as well as other types of spatiotemporal varieties. These structures might be locally generated or convected from elsewhere (such as some of the observed structures in the solar wind that might have been originated from the Sun’s surface). Some of these structures may be more stable than the others. Because of the nature of the physics of complexity, it will be futile to attempt to evaluate and/or study the details and stabilities of each of these infinite varieties of structures; although some basic understanding of each type of these structures will generally be helpful in the comprehension of the full complexity of the underlying nonlinear plasma dynamical system. Examples of coherent structures abound in the literature on the theory and observations of nonlinear space plasma processes.

In the solar wind, for example, one form of the dominant coherent structures is the field-aligned Alfvénic flux tubes (Chang, 1999; Bruno *et al.*, 2001), Figure 1a. Figure 1b is a 2D MHD numerical simulation showing the existence of such structures (Wu and Chang, 2000).

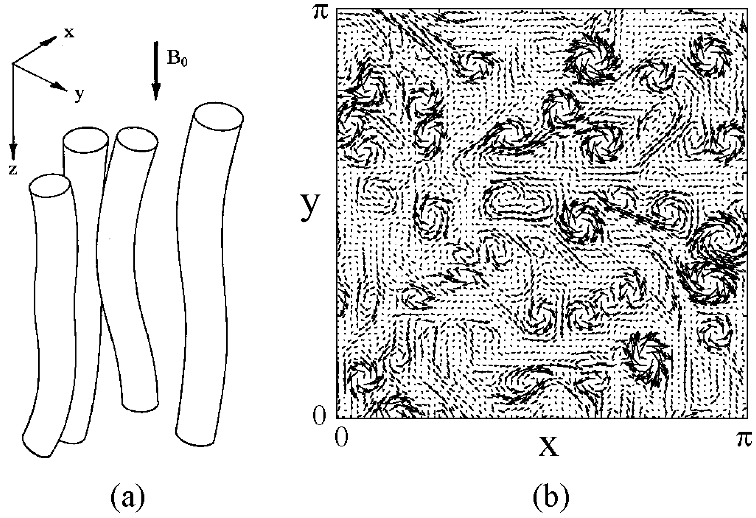


Figure 1. (a) Schematics of field-aligned coherent magnetic structures. (b) 2D MHD numerical simulation of magnetic coherent structures generated by initially randomly distributed current filaments. Shown are the magnetic field vectors.

2.1. Magnetic Reconfiguration

When coherent magnetic flux tubes with the same polarity migrate toward each other, strong local magnetic shears are created. It has been demonstrated by Wu and Chang (2001) that existing sporadic nonpropagating fluctuations will generally migrate toward the strong local shear region. Eventually the mean local energies of the coherent structures will be dissipated into these concentrated fluctuations in the coarse-grained sense and, thereby reconfigure the topologies of the coherent structures of the same polarity into a combined lower local energetic state, eventually allowing the coherent structures to merge locally. And, this merging or "magnetic reconfiguration" process may repeat over and over again among the coherent structures. On the other hand, when coherent structures of opposite polarities approach each other due to the forcing of the surrounding plasma, they might repel each other, scatter, or induce magnetically quiescent localized regions. Such enhanced intermittency at the intersection regions has been observed by Bruno *et al.* (2001) in the solar wind and Consolini *et al.* (2005) in the plasma sheet.

3. Non-Gaussian Probability Distributions

Thus, the dynamical behavior of the interactions of the plasma coherent structures described above is a phenomenon of "complexity." The fluctuations that are induced by such processes are sporadic and localized. Because the coherent structures are numerous and outsized, we expect the fluctuations within the interaction regions of these structures (resonance overlap regions) are generally large and can occur rela-

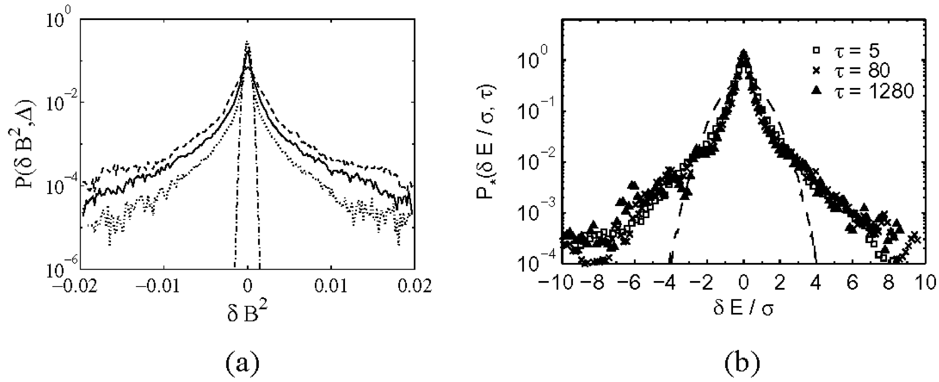


Figure 2. (a) Probability density function $P(\delta B^2, \Delta)$ from the (512×512) 2D MHD simulation for homogeneous turbulence for $\Delta = 2$ (dots), 8 (solid curve), 32 (dashes) units of grid spacing. The dot-dashed curve is the PDF for Gaussian fluctuations. (b) Normalized PDF $P_*(\delta E/\sigma, \tau)$ at $\tau = 5, 80, 1280$ ms. $\delta E \equiv E(t+\tau) - E(t), \sigma^2 = \langle(\delta E)^2\rangle$, and E is an electric field component perpendicular to the geomagnetic field approximately in the geographical north direction in the auroral zone. The dashed line is the PDF for Gaussian fluctuations.

tively often than those would have been expected from a medium of uniformly sized plasma particles. Such statistical characteristic of "*extreme events*" exemplifies the phenomenon of intermittent turbulence in space plasmas. For turbulent dynamical systems with intermittency, the coarse-grained dissipation and transfer of energy (or other relevant scalars and tensors) due to fluctuations deviates significantly from uniformity and varies from scale to scale.

A technique of measuring such degree of intermittency is the study of the departure from Gaussianity the probability distribution functions (PDF) of turbulent fluctuations at different scales. To demonstrate this point, let us again refer to the 2D numerical simulation results described above. For example, we may generate the probability distribution function $P(\delta B^2, \Delta)$ of $\delta B^2(x, \Delta) \equiv B^2(x+\Delta) - B^2(x)$ at a given time t for such simulations, where Δ is the scale of separation in the x-direction. Figure 2a displays the calculated results of $P(\delta B^2, \Delta)$ from the (512×512) 2D simulation for the homogeneous case for several scales Δ. From this figure, we note that the deviation from Gaussianity becomes more and more pronounced at smaller and smaller scales.

Such non-Gaussian probability density functions have been observed in the solar wind (Sorrivo-Valvo *et al.*, 1999; Hnat *et al.*, 2002; Forman and Burlaga, 2003; Jurac, 2003; Leubner, 2004; Leubner and Vörös, 2005), and in the plasma sheet (Angelopoulos *et al.*, 1999; Weygand *et al.*, 2005). In the papers of Leubner (2004) and Leubner and Vörös (2005), nonextensive statistics and long-range interactions were proposed to understand the non-Gaussian shapes of probability density functions in space plasmas. Recently, such probability density functions have also been observed for the time series of electric field fluctuations in the aurora zone (Tam *et al.*, 2005), Figure 2b.

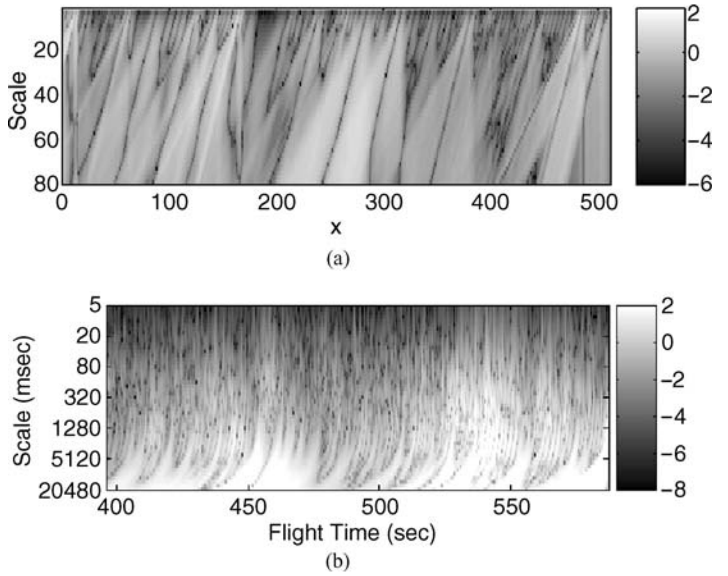


Figure 3. (a) Logarithm of the Haar wavelet power spectrum of the 2D (512 × 512) MHD simulation result for $J_z(x)$ with grid points for a constant value of y at $t = 300$. The x-axis and scales are in units of the grid spacing. (b) Logarithm of the Haar wavelet power spectrum of a section of the time series for one component of the perpendicular electric field fluctuations in the auroral zone.

4. Wavelet Transforms

It will be interesting to study the degrees of intermittency locally at different scales. This can be accomplished using the wavelet transforms. Unlike the Fourier transform which is composed of sinusoidal functions, a wavelet transform generally is composed of localized functions that are capable of unfolding fluctuating fields into space and scale (Farge, 1992). Figure 3a is the logarithm of the power spectrum of the Haar wavelet transform of the current density J_z for the (512×512) 2D MHD homogeneous simulation. We notice that the intensity of the current density is indeed sporadic and varies nonuniformly with scale.

Wavelet transforms have been applied to the study of intermittency in the solar wind by Bruno *et al.* (2001), and the plasma sheet by Consolini and Chang (2001) and Vörös *et al.* (2004). More recently, it has also been applied to the intermittent electric field fluctuations in the auroral zone (Tam *et al.*, 2005), Figure 3b.

5. Multifractals in Intermittent Turbulence

As we have seen above, turbulence in space plasmas generally encompass fluctuations of all varieties and sizes, which interact and propagate throughout the plasma medium. For illustrative purposes, let us visualize some particular fluctuations that have conventional geometrical properties in a three-dimensional Euclidean space. Because of their sporadic and localized nature, it is easy to imagine that they generally cannot fill the full three-dimensional space that they occupy at a given time. Or, say it in another way, the space these fluctuations occupy is only a fraction of the full three-dimensional space. Such geometrical property was popularized by Mandelbrot (1977) when he coined the word, fractals, or fractal geometry.

Actual fluctuations in plasma turbulence generally do not have the conventional geometrical properties. We must then devise some abstract "measure" that characterize the properties of the fluctuations and evaluate their fractal characteristics that may be interpreted with geometrical analogs. Consider, for example, the spatial series of the simulated fluctuations of the strength of the magnetic field, $B(x_i)$ along some constant value of y at time t for the two-dimensional homogeneous MHD turbulence discussed in the previous sections, where $x_i = i\delta$ with $i = 0, 1, 2, \ldots, N$ and δ the length between grid points. From this series, we can construct a spatial series by considering, for example, the absolute value of the fluctuations due to the coarse-grained difference of the square of the strength of the magnetic field between two spatial values $x_i + \Delta$ and x_i with $\Delta = k\delta$, some multiple of δ:

$$\delta B_i^2 = |B^2(x_i + \Delta) - B^2(x_i)| \tag{1}$$

within the spatial interval $X = N\delta$. The simulation is statistically homogeneous over the interval X. Thus, we may calculate the ensemble average of δB_i^2 over the interval X,

$$m(\delta B^2, \Delta) = \langle |B^2(x_i + \Delta) - B^2(x_i)| \rangle \tag{2}$$

and use it as a "measure" for the coarse-grained fluctuations of the simulated spatial series. We can then plot $\log m(\delta B^2, \Delta)$ against $\log \Delta$ for different choices of Δ (or k). If the result approximates a straight line for some range of Δ for small Δ, we can then assign a fractal number $\zeta(\delta B^2)$ to the fluctuations of the strength of the magnetic field as:

$$\zeta(\delta B^2) = \mathrm{d}(\log\, m(\delta B^2, \Delta))/d(\log\, \Delta) \tag{3}$$

for this range of small Δ; Figure 4. In other words, within this range of small Δ; we may represent $m(\delta B^2, \Delta)$ as a power law of Δ with exponent $\zeta(\delta B^2)$. This exponent may be considered as an analog to the classical concept of dimension. It is generally an irrational number and usually cannot be surmised from simple dimensional analysis arguments; thus, it is sort of a "fractal dimension" for the particular choice of "measure" described above.

It is obvious that if another choice of measure is made, the corresponding fractal number for the same spatial series will generally be different. For example, one might wish to look at higher order moments of δB_i^2, the so-called structure functions:

$$m_q(\delta B^2, \Delta) = \langle |B^2(x_i + \Delta) - B^2(x_i)|^q \rangle \tag{4}$$

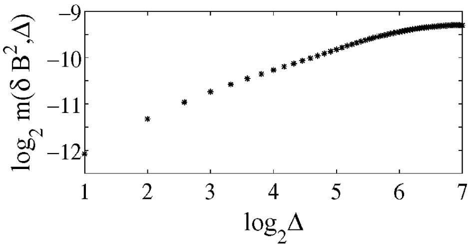


Figure 4. Typical $\log m(\delta B^2, \Delta)$ vs. $\log \Delta$ plot for the 2D MHD homogeneous simulation.

The motivation here is that different moments emphasize different peaks in the fluctuating series. Generally, corresponding to each m_q there will be a fractal exponent ζ_q for small Δ. If $\zeta_q = \zeta_1 q$, then the fractal property of the fluctuating series is fully characterized by the value of ζ_1. Such fluctuations are then said to be "monofractal" or "self-affine", i.e., the fractal characteristics for all the moment orders are similar to each other. For general intermittent turbulence, on the other hand, ζ_q would be a nonlinear function of q, or "multifractal." Such calculations have been carried out for the solar wind (e.g., Marsch and Tu, 1997; Pagel and Balogh, 2001) and elsewhere in the space environment. In practice, such structure function calculations may be performed conveniently for a fluctuating series for any positive value of q; but, unfortunately, will generally diverge for $q < 0$. A more preferred procedure to evaluate the fractal characteristics of different moment orders is the so-called "singularity analysis" described below.

As we have seen from previous sections, turbulence in space plasmas is generally intermittent and therefore probably composed of an admixture of fluctuations of different fractal characteristics. This is true also for the simulated 2D MHD homogeneous turbulence described above. Even though the simulation is homogeneous for the full simulation range, it is nevertheless intermittent at small scales as we have shown in terms of the Probability Distribution Functions (PDF). We may attempt to extract this "multifractal" nature of the simulated result by searching for the "dominant singular behavior" for different moment orders at small scales. A useful technique of accomplishing this was suggested by Halsey *et al.* (1986) and applied to space plasmas by Burlaga (1991), Consolini *et al.* (1996), Lui (2001), Yordonova *et al.* (2004), and Weygand *et al.* (2005). We shall demonstrate this procedure using the results of the two-dimensional homogeneous MHD simulation discussed above. Thus, we define an "incremental measure":

$$\delta\mu_j = |B^2(x_j + \delta) - B^2(x_j)| \tag{5}$$

where δ is the length between grid points. We now subdivide the simulation length into $M = X/\Delta$ segments of length $\Delta = k\delta$ with $X = N\delta$, and calculate the normalized "segmental measure"

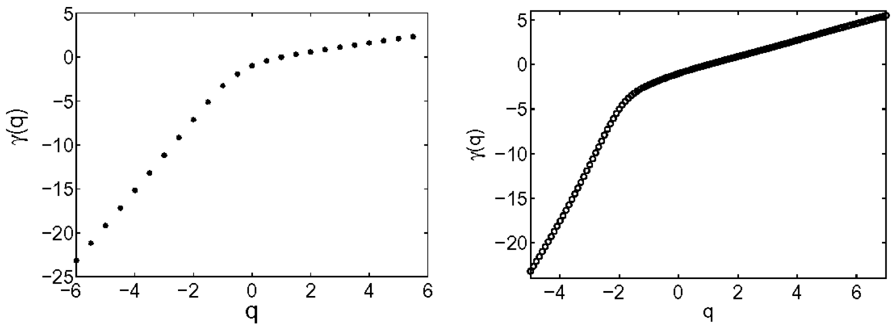
$$\mu_i(\Delta) = \sum_{j=(i-1)k+1}^{ik} \frac{\delta\mu_j}{\mu} \qquad \text{with} \quad \mu = \sum_{j=1}^{N} \delta\mu_j. \tag{6}$$

For convenience, we will generally choose M to be an integer. The normalization of $\mu_i(\Delta)$ is an attempt to represent the segmental measure as a measure of probability. We now form the qth moment order of the coarse-grained probabilities $\mu_i(\Delta)$, traditionally called the "partition function" (Halsey *et al.*, 1986):

$$\Gamma_q(\Delta) = \sum_{i=1}^{M} \mu_i^q(\Delta) \tag{7}$$

where q can be any real number. We shall assume that the dominant singular behavior of $\Gamma_q(\Delta)$ is characterized by a power law in Δ with exponent $\gamma(q)$ for small Δ. In general, for each moment order, $\gamma(q)$ is a different number characterizing the particular fractal behavior of the subset of fluctuations, which dominates the (singular) scaling behavior of that particular moment order; thus, the nomenclature "multifractal."

We have carried out the singularity measure multifractal analysis of the calculated results for the 2D MHD simulation described above. Typical results are shown in Figure 5a. We note from the figure that $\gamma(q)$ is indeed a nonlinear function of q. There are two distinct asymptotic regimes of $q(q< -1$, and q$> +1)$ for which $\gamma(q)$ varies with q essentially linearly and a nonlinear crossover region between the two asymptotes. Thus, the variation of $\gamma(q)$ with q for this case may be understood as the crossover between two monofractals. One is generally limited to the availability of in-situ time series observations when applying these ideas to space plasma turbulence. For solar wind turbulence, it is generally homogeneous at large



(a) (b)

Figure 5. (a) Typical $\gamma(q)$ vs. q plots for the 2D MHD homogeneous simulation at $t = 600$ at small scales. (b) Multifractal partition function exponent $\gamma(q)$ at small scales as a function of the moment order q for the transverse electric field fluctuations in the auroral zone.

scales. And, since the spacecraft speed is much smaller than the solar wind speed, a time series measurement of fluctuations is essentially the same as the spatial series of fluctuations as the solar wind sweeps over the spacecraft. For turbulence in the plasma sheet or the auroral zone, such an assumption (Taylor's hypothesis) must be applied with caution. It is interesting to note that in multifractal analyses of space plasma turbulence in regions where intermittency are expected, similar functional dependence of $\gamma(q)$ on q as above is also generally obtained (Burlaga, 1991; Consolini *et al.*, 1996; Lui, 2002; Yordonova *et al.*, 2004; Weygand *et al.*, 2005).

We have applied the above ideas to the time series of the transverse electric field fluctuations observed in the auroral zone. The results are shown in Figure 5(b).

We note the striking similarity between Figure 5(a) and 5(b). Again, for Figure 5(b) there are two linear asymptotic regimes: one for $q < -2$ and one for $q > 0$ Thus, the multifractal characteristics for the transverse electric field fluctuations for small scales in the auroral zone can again be interpreted as the crossover between two monofractals.

6. Summary and Future Directions

We have provided a modern description of dynamical complexity relevant to the phenomenon of intermittent turbulence in space plasmas. Concepts of probability distribution functions, wavelet transforms, intermittent turbulence, multifractal processes, and extreme events are supplied as answers to a set of frequently asked questions (FAQ).

We now ask what lies ahead by posing a new set of inquisitive queries.

- What characterizes the dynamic multi-scale and cross-scale interactions of the macro- and kinetic coherent structures in magnetized plasma environments?
- What dynamical processes can lead to the multifractal, self-organized critical (SOC) states, and the dynamical crossover effects that permeate the stochastic behavior of space plasma processes and produce the type of observed intermittent turbulence in the space environment?
- What are the scaling properties of the probability distribution functions in the various regimes of the solar wind, magnetosphere, the plasma sheet and the magnetosphere-ionosphere coupling region? And, how do we develop large scale numerical models to investigate these effects?
- How do we develop models that can ultimately lead us to the quantitative prediction of and comparison with probability distribution functions in space plasmas?
- How do we develop theoretical and numerical models that can be used to predict quantitatively the local intermittency and multifractal characteristics

of turbulence phenomena in space plasmas, which can be compared with actual observations?
- What are the effects of intermittency of plasma fluctuations on the energization process of charged particles in the space environment?
- How do intermittent turbulence and multifractal processes influence the global phenomena of space plasmas?

Some preliminary answers to these questions as well as a discussion of the dynamic renormalization group may be found in a recent paper by Chang *et al.* (2004). The readers are encouraged to consult the contents of this treatise and its extensive reference list for further directions of this emerging new field of research in space plasmas.

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References

Angelopoulos, V., Mukai, T., and Kokubun, S.: 1999, *Phys. Plasmas* **6**, 4161.
Bak, P., Tang, C., and Wiesenfeld, K.: 1987, *Phys. Rev. Lett.* **59**, 381.
Baker, D., Klimas, A.J., McPherron, R.L., and Büchner, J.: 1990, *Geophys. Res. Lett.* **17**, 41.
Bruno, R., Carbone, V., Veltri, P., Pietropaolo, E., and Bavassano, B.: 2001, *Planet. Space Sci.* **49**, 12.
Burlaga, J.F.: 1991, *Geophys. Res. Lett.* **18**, 69.
Chang, T.: 1992, *IEEE Trans. Plasma Sci.* **20**, 691.
Chang, T.: 1999, *Phys. Plasmas* **6**, 4137.
Chang, T., Tam, S.W.Y., and Wu, C.C.: 2004, *Phys. Plasmas* **11**, 1287.
Consolini, G.: 1997, In *Cosmic Physics in the Year 2000*, Aiello, S., Lucci, N., Sironi, G., Treves, A., and Villante, U. (eds.), Soc. Ital. di Fis., Bologna, Italy, p. 123.
Consolini, G. and Chang, T.: 2001, *Space Sci. Rev.* **95**, 309.
Consolini, G., Marcucci, M.F., and Candidi, M.: 1996, *Phys. Res. Lett.* **76**, 4082.
Consolini, G., Chang, T., and Lui, A.T.Y.: 2005, In *Nonequilibrium Phenomena in Plasmas*, Sharma, A.S., and Kaw, P.K. (eds.), Springer, The Netherlands, p. 51.
Forman, M. and Burlaga, L.F.: 2003, In *Solar Wind Ten, AIP Conference Proceedings,* Velli, M., Bruno, R., and Malara, F. (eds.), Vol. 679, Melville, NY, p. 554.
Farge, M.: 1992, *Annu. Rev. Fluid Mech.* **24**, 395.
Halsey, T., Jensen, M., Kadanoff, L., Procaccia, I., and Shraiman, B.: 1986, *Phys. Rev. A* **33**, 1141.
Hnat, B., Chapman, S. C., Rowlands, G., Watkins, N.W., and Farrel, W.M.: 2002, *Geophys. Res. Lett.* **29**(10), doi10.1029/2001GL014587.
Jurac, S.: 2003, private communication.
Klimas, A.J., Baker, D.N., Roberts, D.A., Fairfield, D.H., and Büchner, J.: 1992, *J. Geophys. Res.* **97**, 12253.

Leubner, M.P.: 2004, *Phys. Plasmas* **11**, 1308.
Leubner, M.P. and Vörös, Z.: 2005, *Astrophys. J.* **618**, 547.
Lorenz, E.N.: 1963, *J. Atmos. Sci.* **20**, 130.
Lu, E.T. and Hamilton, R.J.: 1991, *Astrophys. J.* **380**, L89.
Lui, A.T.Y.: 1991, *J. Geophys. Res.* **96**, 1849.
Lui, A.T.Y.: 2002, *J. Atmos. Solar-Terres. Phys.* **64**, 125.
Mandelbrot, B.: 1977, *Fractals, Forms, Chance and Dimension*, Freeman and Co., San Francisco.
Marsch, E. and Tu, C.Y.: 1997, *Nonlin. Process. Geophys.* **4**, 101.
Pagel, C. and Balogh, A.: 2001, *Nonlin. Process. Geophys.* **8**, 313.
Roberts, D.A., Baker, D.N., Klimas, A.J., and Bargatze, L.F.: 1991, *Geophys. Res. Lett.* **18**, 151.
Shan, L.H., Goertz, C.K., and Smith, R.A.: 1991, *Geophys. Res. Lett.* **18**, 1647.
Sharma, A.S., Vassiliadis, D., and Papadopoulos, K.: 1993, *Geophys. Res. Lett.* **20**, 335.
Sorriso-Valvo, L., Carbone, V., Veltri, P., Consolini, G., and Bruno, R.: 1999, *Geophys. Res. Lett.* **26**, 1801.
Tam, S.W.Y., Chang, T., Kintner, P.M., and Klatt, E.: 2005, *Geophys. Res. Lett.* **32**(10), L05109, doi:10.1029/2004GL021445.
Vörös, Z., Baumjohann, W., Makamura, R., Runov, A., Volwerk, M., and Chang, T.L.: 2004, *Phys. Plasmas* **11**, 1333.
Weygand, J., Kivelson, M.G., Khurana, K.K., Schwarzl, H.K., Thompson, S.M., McPherron, R.L., et al.: 2005, *J. Geophys. Res.* **10**, doi10.1029/2004JA010581.
Wu, C.C. and Chang, T.: 2000, *IEEE Trans. Plasma Sci.* **28**, 1938.
Wu, C.C. and Chang, T.: 2001, *J. Atmos. Solar-Terres. Phys.* **63**, 1447.
Yordonova, E., Grzesiak, M., Wermik, A.W., Popielawska, B., and Stasiewicz, K.: 2004, *Ann. Geophys.* **22**, 243.