"Complexity" Induced Plasma Turbulence in Coronal Holes and the Solar Wind

Tom Chang

Center for Space research, Center for Theoretical Geo/Cosmo Plasma Physics
Massachusetts Institute of Technology, Cambridge, MA 02139 USA

Abstract. Global kinetic wave-particle interaction theories [1-4] have demonstrated that plasma turbulence emanating from coronal holes may efficiently accelerate the solar wind to observed characteristics and speeds [5,6]. The origin of such turbulence is not known although it is sometimes attributed to the small-scale reconnection processes. We demonstrate that sporadic, localized creation of magnetic coherent structures that arose from Alfvénic resonances can produce the type of turbulence as generally expected. When conditions are favorable, the coherent structures can merge, interact, bifurcate, convect and evolve into a "complex" state of forced and/or self-organized criticality (FSOC) leading to a broad-band power-law spectrum of plasma fluctuations of all scales [7,8]. Phenomenological models are constructed to represent the dynamic fluctuations near the coronal hole base. Dynamic renormalization-group calculations yield values of the scaling exponents that seem to agree with previously conjectured estimates [9]. This result is equally relevant to the basic understanding of the fraction of the non-propagating pseudo-2D plasma fluctuations that are prevalent in the solar wind [10]. A brief discussion on resonant heating of solar wind ions by non-propagating turbulent fluctuations that do not satisfy wave dispersion relations is also included [11,12].

INTRODUCTION

"Complexity" has become a hot topic in nearly every field of modern physics. In this short communication, we demonstrate that the sporadic and localized interactions of magnetic coherent structures are the origin of "complexity" in coronal hole and non-propagating pseudo-2D solar wind turbulence. The intermittent localized interactions, which generate the anomalous diffusion, transport and evolution of the macroscopic state variables of the overall dynamical system, may be modeled [13] by a triggered localized chaotic growth equation of a set of relevant order parameters. Such processes would generally pave the way for the global system to evolve into a "complex" state of long-ranged interactions of non-propagating fluctuations, displaying the phenomenon of forced and (or) self-organized criticality (FSOC). Dynamic renormalization-group analyses (DRG) for simple phenomenological models mimicking coronal hole turbulence yield reasonable ω-scaling exponents for the correlation of the transverse flux function and the trace of the transverse magnetic correlation tensor. These results and their generalizations should also have relevance in addressing the fraction of the non-propagating pseudo-2D turbulence that are generally detected in the solar wind [10]. We also address the concept of resonant energization of solar wind ions by non-propagating turbulent fluctuations that do not satisfy wave dispersion relations [11,12].

PLASMA RESONANCES AND COHERENT STRUCTURES

Most field theoretical discussions begin with the concept of propagation of waves. For example, in the MHD formulation, one can combine the basic equations and express them in the following propagation forms:

$$\rho d\mathbf{V} / dt = \mathbf{B} \cdot \nabla \mathbf{B} + \cdots \tag{1}$$

$$d\mathbf{B} / dt = \mathbf{B} \cdot \nabla \mathbf{V} + \cdots \tag{2}$$

where the ellipses represent the effects of the anisotropic pressure tensor, the compressible and dissipative effects, and all notations are standard. Equations (1,2) admit the well-known Alfvén waves. For such waves to propagate, the propagation vector **k** must contain a field-aligned component, i.e., $\mathbf{B} \cdot \nabla \to i\mathbf{k} \cdot \mathbf{B} \neq 0$. However, at sites where the parallel component of the propagation vector vanishes (i.e., at the resonance sites), the fluctuations are localized. Around these resonance sites (usually in the form of curves), it may be shown that the fluctuations are held back by the background magnetic field, forming Alfvénic coherent structures in the form of flux tubes or current filaments [8,11].

Generally, there exist various types of propagating modes (whistler modes, lower hybrid waves, etc.) in a continuum plasma. Thus, we envision a corresponding number of different types of plasma resonances and associated coherent structures that typically characterize the dynamics of the plasma medium under the influence of a background magnetic field. These coherent structures will wiggle, migrate, deform and undergo different types of motions under the influence of the local plasma and magnetic topology. In the next section, we will consider how the coherent structures can interact and produce the type of intermittency generally observed in dynamical plasmas.

INTERACTIONS OF COHERENT STRUCTURES

When filamentary current structures of the same polarity migrate toward each other, strong current sheets are generated [8,14,15]. As the electrons travel across the magnetic field lines, they would excite whistler fluctuations. Now, in analogy to the Alfvén resonances, singularities of $k_{\|} = \mathbf{k} \bullet \mathbf{B} = 0$ generally can develop at which whistler fluctuations cannot propagate. These “whistler resonances” can provide the nuclear sites for the emergence of coherent whistler structures, which is the analog of the coherent Alfvénic structures but with much smaller scales. The intermittent turbulence resulting from the intermixing and interactions of the coherent whistler and small scale Alfvénic structures in the intense current sheet region can then provide the coarse-grain averaged dissipation that allows the larger Alfvénic coherent structures to merge, interact, or breakup [8].

CORONAL HOLE TURBULENCE

The above intermittency description for non-propagating plasma turbulence may be modeled by the combination of a localized chaotic functional growth equation for a set of relevant order parameters and a functional transport equation for the control parameters [13]. The resulting transport processes will generally be sporadic and anomalous [16]. Due to the limitation of space in this short communication, we shall consider instead below two simple phenomenological models, which may have some relevance to coronal hole turbulence.

To evaluate the scaling properties of these kinetic models near complex states of long-ranged correlations (FSOC), we shall employ the functional method of classical path integrals and the dynamic renormalization-group (DRG) to be outlined below [17,18].

Model I

Assuming that the parallel mean magnetic field B_0 is sufficiently strong and the magnetic fluctuations dominate in the transverse directions, we introduce the flux function ψ for the non-propagating transverse fluctuations as follows,

$$\mathbf{B} = \mathbf{e}_z \times \nabla \psi + B_0 \mathbf{e}_z \tag{3}$$

This insures the magnetic field to be divergence free.

The coherent structures for such a system are generally flux tubes approximately aligned in the mean parallel direction [11]. Conservation of helicity indicates that the integral of ψ over a flux tube is approximately constant. Instead of invoking the standard reduced MHD formalism, here we simply consider ψ as a dynamic order parameter. As the flux tubes merge and interact, they may correlate over long distances, which, in turn, will induce long relaxation times near FSOC [7]. Let us assume that the base of the coronal hole is sufficiently broad compared to the cross sections of the coherent structures (or flux tubes), such that we may invoke homogeneity and assume the dynamics to be independent of boundary effects. We may then model the dynamics of flux tube mergings and interactions, in the crudest approximation, in terms of the following order-disorder intermittency equation:

$$\partial \psi_k / \partial t = -\Gamma_k \partial F / \partial \psi_{-k} + f_k \tag{4}$$

where ψ_k are the Fourier components of the flux function, Γ_k an analytic function of k^2, f_k a random noise which includes all the other effects that had been neglected in this crude model, and $F(\psi_k, k)$ the state function.

Model II

In the above model, we have neglected both the effects of diffusion and convection. We next construct a phenomenological model that includes the transport of cross-field diffusion. We now assume the state function to depend on the flux function ψ and the local "pseudo-energy" measure ξ. Thus, in addition to the dynamic equation (4), we now also include a diffusion equation for ξ. In Fourier space, we have

$$\partial \xi_k / \partial t = -Dk^2 \partial F / \partial \xi_{-k} + h_k \tag{5}$$

where ξ_k are the Fourier components of ξ, $D(k)$ is the diffusion coefficient, the state function is now $F(\psi_k, \xi_k, k)$, and h_k is a random noise. By doing so, we separate the slow transport due to diffusion of the local "pseudo-energy" measure ξ from the noise term of (4).

We shall now proceed to study the complexity and FSOC that can arise from the critical dynamics of these phenomenological models.

DYNAMIC RENORMALIZATION-GROUP ANALYSIS

For nonlinear stochastic systems near criticality, the correlations among the fluctuations of the random dynamical fields are extremely long-ranged and there exist many correlation scales. The dynamics of such systems are notoriously difficult to handle either analytically or numerically. On the other hand, since the correlations are extremely long-ranged, it is reasonable to expect that the system will exhibit some sort of invariance under scale transformations. A powerful technique that utilizes this invariance property is the technique of the dynamic renormalization-group (17,18, and references contained therein). As it is described in these references, based on the path integral formalism, the behavior of a nonlinear stochastic system far from equilibrium may be expressed in terms of a "stochastic Lagrangian L". Then, the renormalization-group (coarse-graining) transformation may be formally expressed as:

$$\partial L / \partial \ell = RL \qquad (6)$$

where R is the renormalization-group transformation (coarse-graining) operator and ℓ *is* the coarse-graining parameter for the continuous group of transformations. It will be convenient to consider the state of the stochastic Lagrangian in terms of its parameters $\{P_n\}$. Equation (6), then, specifies how the Lagrangian, L, flows (changes) with ℓ in the affine space spanned by $\{P_n\}$.

Generally, there exists a number of fixed points (singular points) in the flow field, where $dL / d\ell = 0$. At a fixed point, the correlation length should not be changing. However, the renormalization-group transformation requires that all length scales must change under the coarse-graining procedure. Therefore, to satisfy both requirements, the correlation length must be either infinite or zero. When it is at infinity, the system is by definition at criticality. The alternative trivial case of zero correlation length will not be considered here.

To study the stochastic behavior of a nonlinear dynamical system near a particular criticality, we can linearize the renormalization-group operator R about it. The mathematical consequence of this approximation is that, close to criticality, certain linear combinations of the parameters that characterize the stochastic Lagrangian L will correlate with each other in the form of power laws. This includes, in particular, the (k, ω), i.e. mode number and frequency, spectra of the correlations of the various fluctuations of the dynamic field variables. In addition, it can be demonstrated from such a linearized analysis that generally only a small number of (relevant) parameters are needed to characterize the stochastic state of the system near criticality (i.e., low-dimensional behavior; see [7]).

We have performed dynamic renormalization-group (DRG) analyses for the two kinetic models described in the previous section. We note that under the DRG transformation, the correlation function C of ψ_k should scale as:

$$e^{a_c \ell} C(k, \omega) = C(k e^{\ell}, \omega e^{a_\omega \ell}) \qquad (9)$$

where ω is the Fourier transform of the time *t*, ℓ the renormalization parameter as defined in the previous section, and (a_c, a_ω) the correlation and dynamic exponents. Thus, $C / \omega^{a_c / a_\omega}$ is an absolute invariant under the DRG, or $C \sim \omega^{-\lambda}$, where $\lambda = -a_c / a_\omega$. DRG analysis for Model I with Gaussian noise yields the value of λ approximately equal to 2.0.

DRG analyses performed for Model II for Gaussian noises for several approximations yield the value for λ to be approximately equal to 1.88 - 1.66.

Interestingly, for both models, DRG calculations give a value -1.0 for the ω-exponent for the trace of the transverse magnetic correlation tensor. Matthaeus and Goldstein [9] had suggested that such an exponent might represent the superposition of discrete structures emerge from the solar convection zone. We now turn our attention to the solar wind.

PSEUDO-2D NON-PROPAGATING TURBULENCE IN THE SOLAR WIND

Although the above discussions have been addressed to the turbulence to be expected in the coronal hole base, these results and their generalizations should also have direct bearing on the observed pseudo-2D non-propagating turbulence [10] and flux tubes [19] in the solar wind. It is expected that as the turbulent fluctuations emanate from the coronal hole base, some of the fluctuations will be mode-converted either linearly or nonlinearly into propagating Alfvén wave turbulence. Eventually, as the flux tubes in the open field lines interact, some of the wave turbulence would again be re-converted to non-propagating pseudo-2D fluctuations. The resulting fraction of the non-propagating turbulence can again be addressed by the ideas introduced in this short communication. Generally the magnetic fluctuations are composed of both the propagating and non-propagating components as originally pointed out by Belcher and Davis [20]. These ideas explain why generally the Alfvén ratio in the solar wind is less than one [21]. We demonstrate in the next section that non-propagating turbulent fluctuations can be an important contribution to the resonant energization of solar wind ions.

ENERGIZATION OF IONS

The non-propagating fluctuations discussed above entail a broadband power-law spectrum $\varepsilon(\mathbf{k},\omega)$ of the transverse electric field fluctuations. Because of the broad band nature of the fluctuations, they can provide continuous resonant energization to the solar wind ions. If the process is due to resonant phase-space diffusion, the perpendicular diffusion coefficient to the lowest order would then simply be [12]

$$D_{\perp} = (q/m)^2 \int\int d\omega d\mathbf{k}\, \varepsilon(\mathbf{k},\omega) R(\mathbf{k},\omega,\mathbf{v}) \qquad (7)$$

where (q, m, **v**) are the ion charge, mass, and velocity, respectively, and $R(\mathbf{k},\omega,\mathbf{v})$ is the resonance function that accommodates the required Doppler shifts for resonance interactions, coherence conditions, and stochastic broadening of interactions among the ions and fluctuations. We note that, for resonant energization of the ions, the turbulent $(\mathbf{k},\omega)$-spectra do not need to satisfy any dispersion relations as the fluctuations are generally non-propagating.

CONCLUSION

We have provided a theory of plasma turbulence in the coronal holes and solar wind in terms of complexity generated non-propagating fluctuations. Such non-propagating fluctuations are characterized by the mergings and interactions of coherent structures (generally in the form of flux tubes aligned in the parallel direction of the mean magnetic field) that arose from Alfvénic plasma resonances. Simple phenomenological kinetic models are introduced to characterize such dynamic interactions. Dynamic renormalization-group (DRG) analyses near forced and/or self-organized criticality (FSOC) for these models yield the scaling law $C \sim \omega^{-\lambda}$, where the exponent λ is estimated to be 2.0 and 1.88 - 1.66, and an ω-exponent for the trace of the transverse magnetic correlation tensor approximately equal to -1.0.

These ideas are equally applicable to the solar wind turbulence where non-propagating pseudo-2D fluctuations are commonly observed. The resulting broadband spectrum of the electric field fluctuations can conveniently energize solar wind ions without the fluctuations satisfying any wave dispersion relations.

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