SPECIAL TOPIC

Self-organized criticality, multi-fractal spectra, sporadic localized reconnections and intermittent turbulence in the magnetotail

Tom Chang[a)]
Center for Space Research, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 14 April 1999; accepted 27 May 1999)

It has been suggested that the dynamics of the Earth's magnetotail may be described by the stochastic behavior of a nonlinear dynamical system near forced and/or self-organized criticality (SOC). It has been further argued that multiscale intermittent turbulence of overlapping plasma resonances is the underlying physics that can lead to the onset and evolution of substorms. Such a description provides a convenient explanation of the localized and sporadic nature of the reconnection signatures and fractal spectra that are commonly observed in the magnetotail region. These concepts provide a new paradigm for the understanding of the ever-elusive phenomenon of magnetic substorms. In this review, we describe some of the basic physical concepts and mathematical techniques (such as the dynamic merging of coherent structures, nonclassical nonlinear instability, path integrals, the theory of the renormalization-group, low-dimensional chaos, self-similarity and scaling, fractals, coarse-grained helicity and symmetry breaking) that play a central role in the development of these new ideas. © *1999 American Institute of Physics.* [S1070-664X(99)02611-7]

I. INTRODUCTION

Experimental observations[1,2] indicate that the magnetotail dynamics associated with substorms are intermittent. A model involving the generation, dispersing, and merging of multiscale coherent plasma structures has been suggested recently[3–7] to address the implications of such observations. In this model, the dynamics of the magnetotail has been related to the stochastic behavior of a nonlinear dynamical system near forced and/or self-organized criticality (SOC) and the onset of substorm to an associated global nonclassical, nonlinear instability.

In addition, it has been demonstrated[8–13] that certain characteristics of magnetotail dynamics could be modeled by deterministic chaos of low-dimensional nonlinear systems with fractal characteristics. Using the concepts of the dynamic renormalization group, it can be demonstrated[3,4] that nonlinear stochastic systems near forced and/or self-organized criticality (SOC) such as the hypothesized dynamic states of the Earth's magnetotail can exhibit such low-dimensional and fractal behavior.

Both the physical concepts and mathematical techniques associated with these current ideas are nontraditional. It is the purpose of this review paper to provide an introductory background for these fundamental concepts. Only physical and topological descriptions will be provided. The readers are referred to the various original papers for further in-depth study of these emerging concepts and mathematical as well as simulation techniques.

[a)]Also at International Space Science Institute, CH-3012, Bern, Switzerland. Electronic mail: tsc@space.MIT.edu

The organization of this paper is as follows. In Sec. II, we provide the framework for an intermittent turbulence model of sporadic localized reconnections for the magnetotail. The model incorporates a multitude of cross-scale interactions [kinetic merging and magnetohydrodynamic (MHD) mixing] within the entire dynamic region. We will demonstrate that such a dynamic process will generally require the existence of an underlying three-dimensional magnetic field geometry, and that coherent flux structures and localized merging can set in near sites of Alfvén resonances.

In Sec. III, we digress and discuss the implications of the individual localized merging of coherent structures. We suggest that these are the localized reconnection signatures observed by spacecraft flying through the neutral sheet region of the magnetotail and that they are the origins of the observed ''bursty bulk flows.'' [1] The concept of coarse-grained helicity will also be introduced.

In Sec. IV, it is suggested that the onset of substorms is due to the appearance of a global, nonclassical, nonlinear instability signaled by the enhanced mixing and merging of the coherent structures. The ensuing substorm dynamics would then be shown to be related to the phenomenon of forced and/or self-organized criticality.

For nonlinear stochastic systems near criticality, the correlations among the fluctuations of the random dynamic fields are extremely long-ranged and there exist many correlation scales. Using the concepts of path integrals and renormalization group,[14] we demonstrate in Sec. V that such dynamical systems will acquire the phenomenon of anomalous dimensions.[3,4] We then give a connection between the fractal dimensions of finite dimensional chaotic systems and the

1070-664X/99/6(11)/4137/9/$15.00

anomalous dimensions of the critical state of an infinite-dimensional dynamical system far from equilibrium; thus, providing a rationale for the recent low-dimensional studies for substorm dynamics.[8–13]

In Sec. VI, we discuss the concepts of scaling, similarity, and fractals that emerge naturally from the renormalization-group theory of critical dynamics.[14] We discuss the importance of the fluctuation spectra for the inhomogeneous, anisotropic intermittent turbulent states of the magnetotail. The concept of multi-fractals is introduced in Sec. VII and related to the scaling regions of the spectra for coherent, MHD and kinetic states. The concept of crossover and symmetry breaking are introduced to interpret the dynamic spectra.

The paper concludes, in Sec. VIII with a Summary, and an Appendix on the derivation of the stochastic path integral for dynamic systems far from equilibrium.

As a guide for the reader of this review, Sec. V on the application of the dynamic renormalization-group and the Appendix on the path integral formalism probably contain passages that may appear difficult to comprehend during a first reading. These passages may be bypassed unless the reader is interested in performing specific nonlinear stochastic calculations near criticality from first principles. This paper is not intended as a review of the classical concepts of magnetic reconnection and substorms. Excellent reviews on such topics already exist in the literature.

II. STOCHASTIC MERGING AND MIXING OF COHERENT STRUCTURES

Most of the individual localized merging processes in the Earth's magnetotail occur at intermediate or microscales.[1,2] The dimensions of the full dynamic domain that is responsible for the transferring of energy and momentum from the solar wind to the Earth's magnetotail, on the other hand, generally involve time and spatial scales much larger than those characterized by the microscopic plasma parameters. Thus, the transport processes at the two ends of the dynamic spectrum can involve characteristic parameters differ by orders of magnitude, suggesting that the underlying dynamics of the magnetotail is intrinsically multiscale. To describe the overall energy exchange, it will be necessary to consider the gross mixing of fluctuations at macroscopic scales, while the merging of coherent structures near the ''neutral sheet'' are affected generally by kinetic effects. It is the interplay of the kinetic, intermediate and MHD scale fluctuations and the relevant methodology required to address such questions that form the central theme of this paper.

Thus, we assume that the dynamics of the plasma medium is primarily characterized by the basic MHD variables with an anisotropic pressure tensor. To bring in some of the possible kinetic effects, we can, for example, relate the pressure tensor to the particle distribution functions in terms of the appropriate moments.

Assuming, for the moment, the collisional effects are negligible, the basic equations are then,

$$\partial\rho/\partial t+\rho\nabla\cdot\mathbf{V}=0, \tag{1}$$

$$\rho\, d\mathbf{V}/dt=-\nabla\cdot\mathbf{P}+\mathbf{j}\times\mathbf{B}, \tag{2}$$

$$\nabla\cdot\mathbf{B}=0, \tag{3}$$

$$\partial\mathbf{B}/\partial t+\nabla\times\mathbf{E}=0, \tag{4}$$

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=0, \tag{5}$$

$$\mathbf{j}=\nabla\times\mathbf{B}, \tag{6}$$

where all notations are standard and the pressure tensor is given by

$$\mathbf{P}=\sum_i m_i\int(\mathbf{v}-\mathbf{V})(\mathbf{v}-\mathbf{V})f_i(\mathbf{v})d\mathbf{v}, \tag{7}$$

and the particle distribution functions $f_i(\mathbf{x,v},t)$ satisfy the Vlasov equations.

Standard arguments lead to the following equations of induction and motion:

$$\partial\mathbf{B}/\partial t=(\mathbf{B}\cdot\nabla)\mathbf{V}+\cdots, \tag{8}$$

$$\rho\, d\mathbf{V}/dt=(\mathbf{B}\cdot\nabla)\mathbf{B}+\cdots, \tag{9}$$

where the ellipsis represent the compressible and anisotropic pressure terms. Generally, of course, dissipative terms must be included on the right-hand sides of Eqs. (8) and (9) as well. It is clear from above that one of the wave modes allowed by these equations is the Alfvén wave. For such modes to propagate, the propagation vector $\mathbf{k}$ must contain a field-aligned component, i.e., $\mathbf{B}\cdot\nabla\rightarrow i\mathbf{k}\cdot\mathbf{B}\neq 0$. However, at sites where the parallel component of the propagation vector vanishes, $k_\parallel=\mathbf{k}\cdot\mathbf{B}=0$, energies are localized and the field lines may be distorted effortlessly. These singularities (points, curves or surfaces) at which $k_\parallel=0$ are ''Alfvén resonances.'' As it will be demonstrated below, the existence of Alfvén resonances will lead to the formation of nearly non-propagating and essentially closed macroscopic magnetic structures. Because of the presence of the pressure tensor term in Eq. (9), there exists also the possibility of other macroscopic as well as kinetic resonances.

Coarse-grained helicity. We now consider the magnetic field structures near the Alfvén resonances. For an ideal MHD system, any physically acceptable magnetic field must satisfy Eq. (3). Also, any variation of the field away from the initial value must satisfy the constraints given by Eqs. (4) and (5). Taylor[15] demonstrated that Eqs. (4) and (5) may be replaced by an infinite set of integral constraints involving the *helicity* K, such that

$$K=\int_V \mathbf{A}\cdot\mathbf{B}\, dV \tag{10}$$

is an invariant for any volume V enclosed by a flux surface, where $\mathbf{A}$ is the vector potential. It can be shown that as the system relaxes to its minimum energy state satisfying the helicity conservation constraints, the magnetic structure will be in a force-free state, i.e., $\mathbf{j}\times\mathbf{B}=0$.

Taylor's conjecture. Let us now consider our present situation at hand. We are interested in the more realistic situation that characterizes the dynamics of the magnetotail where the plasma is slightly dissipative and in addition, there are stochastic macroscopic (as well as microscopic) fluctua-

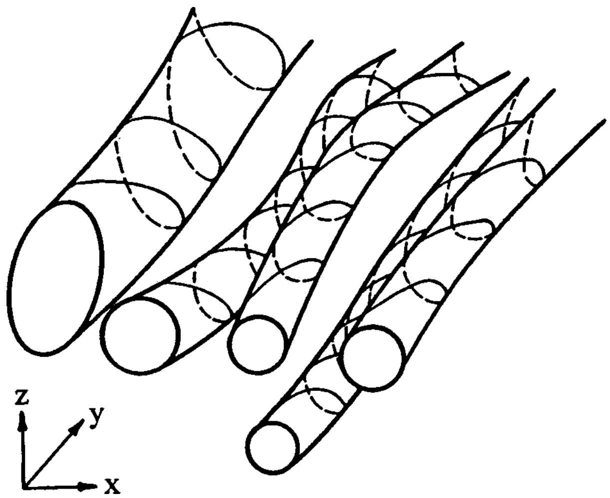

FIG. 1. Twisted flux tubes near the neutral sheet region.

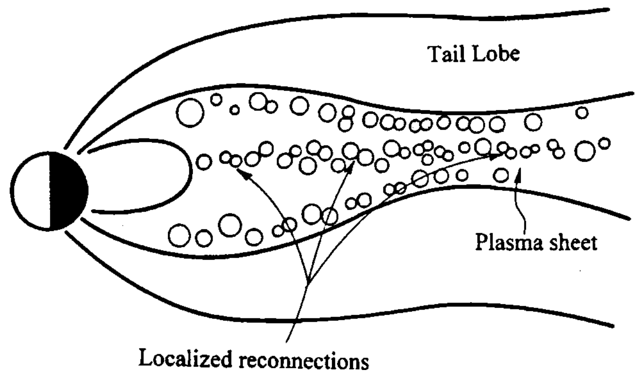

FIG. 2. Cross-sectional view of sporadically distributed flux tubes in the plasma sheet.

tions. The dissipation and stochasticity will allow the field lines to merge, mix, and break. It is obvious that it no longer makes sense to discuss the topology of individual field lines. Nevertheless, it was suggested by Taylor[15] that when the volume integral for Eq. (10) is taken over the ''stochastic region,'' the coarse-grain averaged helicity in a relaxed state will be essentially conserved. This indicates that when considering the stochastic domain, the average magnetic structure will again be essentially force free, with $\mathbf{j}\times\mathbf{B}=0$, where $\mathbf{j}$ and $\mathbf{B}$ are now to be interpreted as the mean current and magnetic field, respectively. This result can also be arrived at using the clump theory of MHD turbulence.[16]

Let us now apply this idea to the sheared magnetic field geometries that are generally found in the ''neutral sheet'' region of the magnetotail. The nearly force-free condition for the coarse-grain averaged coherent structures would then orient themselves more-or-less in the cross-tail current direction in the form of twisted flux tubes, Fig. 1.

As these coherent structures migrate toward each other, they will merge and form new coherent structures. Depending on the polarities and intensities of the currents that orient these flux tubes, the resulting coherent structures will be either larger or smaller than the original individual structures. The final states of the new coherent structures will again be essentially force-free in the coarse-grained sense. As these new structures are generated, new MHD fluctuations are produced; and thereby spontaneously set up new resonance sites. Thus, an interesting scenario of intermittent turbulent mixing, diffusing, and merging sets in. This type of intermittent turbulence is anisotropic, inhomogeneous and multiscale in the magnetotail (Fig. 2). In the following, we shall digress and discuss the individual localized merging processes.

III. LOCALIZED RECONNECTIONS

In the preceding section, we argued that coherent flux tubes tend to form and migrate in sheared magnetic fields similar to the phenomenon generally observed in the ''neutral sheet'' region of the magnetotail. As these structures approach each other, they merge into new coherent structures while simultaneously exciting plasma turbulence and inducing particle acceleration. Let us consider the merging of two coherent structures, viewed in a section normal to the average direction of the cross-tail current, the topologies of the field lines during such a merging process mimic the topology that is generally considered for a classical magnetic ''reconnection'' process (Fig. 3).

Thus, as a spacecraft flies through the neutral sheet region of the magnetotail, there is a finite probability for the instruments on the spacecraft to detect classical-like reconnection signatures. Such signatures can be detected nearly anywhere in the plasma sheet, but more probably in the ''neutral sheet'' region, particularly during substorm times. The duration of interaction of these observed localized merging processes should be the approximate time required for the new relaxed coherent structures to emerge and in general, would be rather sporadic. We suggest that these are the origins of the observed ''bursty bulk flows.''[1,2] The time scale, size, and energy contents involved in these localized merging processes will generally be much smaller than those that are

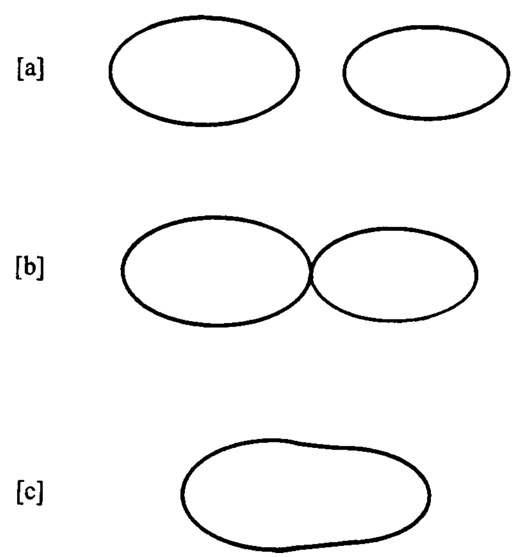

FIG. 3. Cross-sectional view of coherent structures at various stages of merging. (a) Just prior to merging. (b) During the process of merging. (c) Relaxed state after merging.

considered to be relevant for the full dynamics of the magnetotail, particularly during the full duration of the magnetic substorms.

Most of the observed localized reconnection signatures to date seem to indicate that these localized merging processes take place in domain sizes comparable to that of the ion gyroradius, especially during substorm times. Thus, very probably most of these processes will be influenced by microscopic kinetic effects. During these dynamic processes, the ions can probably be assumed to be unmagnetized and the electrons fully magnetized and the plasma nearly collisionless. This, of course, would lead to electron-induced Hall currents. Depending on the underlying magnetic geometry (since these processes can occur at any arbitrary underlying magnetic field configuration), the relevant kinetic instability that can initiate the localized merging (or reconnection) can be any of the many recently suggested microscopic instabilities such as the collisionless tearing instability, or the cross-field two-stream instability.[17] It is very probable that the nonlinear state of merging for each of these localized reconnections again entails the phenomenon of overlapping resonances.[5–7,18] (Now these resonances will arise from the localization of microscopic fluctuations, e.g., the whistler resonances,[5–7] and multiple tearing modes.[19])

IV. NONCLASSICAL, NONLINEAR INSTABILITY

Under favorable conditions (e.g., with the availability of a free energy source such as the enhancement of the cross-tail current due to the change of certain global controlling parameters for the magnetotail), the state of intermittent turbulence discussed in the previous sections may grow by producing more and larger coherent structures and fluctuations as well as new resonance sites (Fig. 2). Eventually, the magnetotail may become unstable. This type of instability, by definition, is genuinely ''nonlinear,'' and usually global in nature (in the sense that the nonlinear dissipation is determined self-consistently through the induced turbulence throughout the medium subject to the global parameters that control the magnetotail dynamics).

For the onset and growth of a classical nonlinear instability, there generally exists a prescribed minimum finite amplitude of disturbance (measured, for example, by the root-mean-square of fluctuations) beyond which the fluctuations and coherent structures can grow provided that there is an available abundance of efficient convertible free energy (Fig. 4). Much attention has been paid to the onset of magnetic substorms. During the onset of a substorm, it has been suggested[3,4] that the nonequilibrium plasma state in the magnetotail is near criticality (similar to the critical point for equilibrium liquid/gas phase transitions). At such a dynamic state (referred to in this paper as a state of forced and/or self-organized criticality, SOC), the effect of the fluctuations themselves can become an important factor in determining the critical threshold of onset. In this situation, the nonlinear instability is no longer described by its classical threshold (Fig. 4).

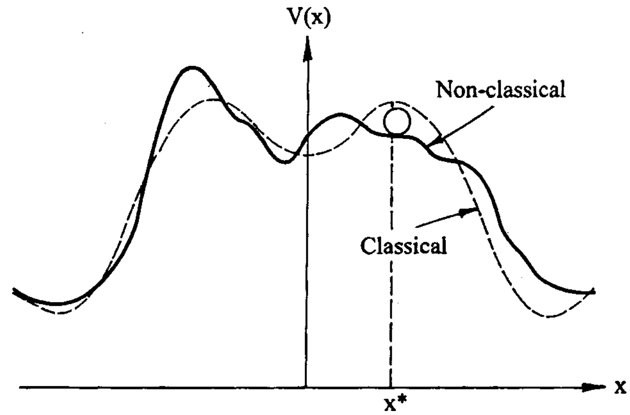

FIG. 4. Marble rolling over a hill. Schematic representation of classical and non-classical, nonlinear instability. x^*: Classical threshold for nonlinear instability.

V. THE DYNAMIC RENORMALIZATION GROUP

In several recent papers,[8–13] it has been suggested that certain substorm characteristics in terms of the AL and/or AE time series could be modeled by deterministic chaos of simple low-dimensional dynamic equations. It may be demonstrated[3,4] that the above results are the consequences of a dynamic system near forced and/or self-organized criticality.

For nonlinear stochastic systems near criticality (such as the situation during the evolution of substorms discussed above), the correlations among the fluctuations of the random dynamic fields are extremely long-ranged and that there exist many correlation scales. The dynamics of such systems are notoriously difficult to handle analytically or numerically. On the other hand, since the correlations are extremely long-ranged, it is reasonable to expect that the system will exhibit some sort of invariance under scale transformations. (For discussions of phase transitions and critical phenomena, the readers are encouraged to consult Stanley.[20]) A powerful technique that utilizes this invariance property is the generalization of a renormalization group transformation procedure originally suggested for static critical phenomena,[21,22] critical dynamics and dynamical systems far from equilibrium.[3,4,14]

As it is discussed in the Appendix, based on the path integral formalism, the behavior of a nonlinear stochastic system far from equilibrium may be described in terms of a ''stochastic Lagrangian L.'' Then, the renormalization-group transformation may be formally expressed as:

$$\partial L/\partial \ell = \mathbf{R}L \tag{11}$$

where $\mathbf{R}$ is the renormalization-group transformation operator, ℓ is the scaling (renormalization) parameter for the continuous group of transformations. It will be convenient to consider the state of the stochastic Lagrangian in terms of the parameters $\{P_n\}$ that characterize it. Equation (11), then, specifies how the Lagrangian, L, flows (changes) with ℓ in the affine space spanned by $\{P_n\}$, Fig. 5.

Generally, there exists a number of fixed points (singular points) in the flow field, where $dL/d\ell = 0$. At a fixed point (e.g., L^* or L^{**} in Figs. 5 and 6), the correlation length

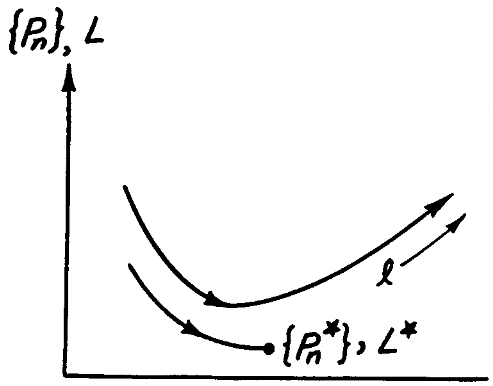

FIG. 5. Schematic representation of renormalization-group flow in the parameter space of the stochastic Lagrangian. Arrows indicate directions of increasing ℓ. There is a fixed point $(\partial L/\partial\ell=0)$ at L^*.

should not be changing. However, the transformation of Eq. (11) insists that all scales (including the correlation length) must change continuously. We therefore conclude that at a fixed point, the correlation length must be either infinite or zero. When the correlation length is infinite, the system is at criticality. The alternative trivial case of zero correlation length will not be considered here.

To study the stochastic behavior of a nonlinear dynamical system near a particular criticality (e.g., the one characterized by L^*), we linearize the renormalization-group operator $\mathbf{R}$ about L^* and obtain

$$\partial L'/d\ell=R_L L' \quad \text{or} \quad dP'_m/d\ell=\Sigma(R_L)_{mn}P'_n, \tag{12}$$

where $L'=L-L^*$, $P'_n=P_n-P^*_n$ and R_L or $(R_L)_{mn}$ is the linearized renormalization-group operator. Diagonalizing Eq. (12), we obtain

$$dV_k/d\ell=\lambda_k V_k \quad \text{or} \quad V_k=V_{k0}\exp(\lambda_k\ell), \tag{13}$$

where $V_{k0}=V_k(\ell=0)$. The λ_k's are the eigenvalues and the V_k's are the eigenvectors or eigen-directions, which are particular combinations of a selected set of the original parameters of the stochastic system. [If the matrix $(R_L)_{mn}$ is asymmetrical, it may not be possible to find a complete set of eigenvectors satisfying Eq. (13). In that case, there may be logarithmic corrections. These discussions are beyond the

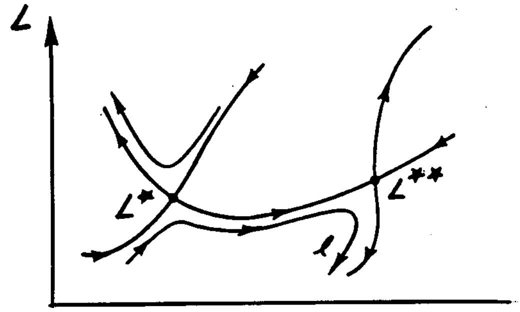

FIG. 6. Schematic representation of renormalization-group flow with two competing fixed points L^* and L^{**}. Arrows indicate directions of increasing ℓ.

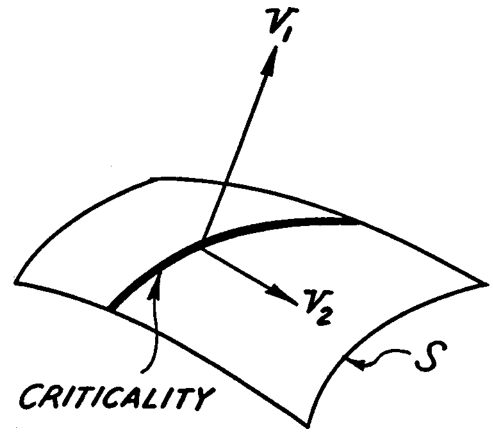

FIG. 7. Geometric representation of relevant scaling directions in an N-parameter affine space. S is an $(N-1)$-dimensional surface of constraint, and the heavy line represents an $(N-2)$-dimensional surface of criticality.

scope of the present treatise.] The Lagrangian then is approximately expressible as a linear combination of its eigenoperators or eigendensities:

$$L'(\ell)=\Sigma V_k(\ell)U_k=\Sigma V_{k0}\exp(\lambda_k\ell)U_k. \tag{14}$$

The eigendensities U_k's are generally linear combinations of a selected set of relevant extensive field variables (such as density) that characterize the stochastic system.

Generally, the eigenvalues λ_k's are real. Thus, after a large number of iterations, i.e., for large ℓ, only those operators with positive λ_k's survive. These surviving eigenvalues are called the ''relevant eigenvalues.'' (We shall avoid discussing the complicated situation when λ_k vanishes.) Therefore, near criticality, the behavior of the stochastic system depends on only those relevant eigenvectors in the parameter space where the eigenvalues λ_k's are positive and the scaling property of the system is given by Eqs. (13) and (14) with $\lambda_k>0$.

Usually, there are only a small number of relevant eigenvalues λ_k's at criticality. This can be understood geometrically as follows. Figure 7 is a schematic representation of the scaling directions in the affine geometry of an N-dimensional parameter space. The $(N\text{-}1)$-dimensional surface, S, represents some sort of constraint on the physical stochastic system and the heavy line represents a surface of $(N\text{-}2)$-dimensions at criticality while V_1 and V_2 represent the two relevant scaling directions. In this case, there can only be two relevant directions emanating away from the critical surface because in affine geometry there is no metric and directions can only be differentiated by ''parallelism.'' The two directions may be understood geometrically as (1) a direction not contained in the surface of constraint and (2) a direction lying in the constraining surface but not contained in the critical surface. In this case, the number of relevant parameters is exactly two, which are represented by the relevant eigen-directions, each of which is a linear combination of certain relevant physical parameters $\{P_n\}$ that characterize the stochastic system. We therefore can claim that the ''dimension'' of the stochastic system near this particular critical state is two, (actually not necessarily exactly two, however, because of the discussions given below).

VI. SCALING, SIMILARITY, AND FRACTALS

It is straightforward to verify from Eq. (13) that the ratio

$$V_i/V_j^{\lambda_i/\lambda_j}=\text{constant} \tag{15}$$

for the relevant eigenvectors V_i and V_j. In other words, they are invariants under the renormalization-group transformation near criticality characterized by a particular fixed point. Therefore, in a log–log plot, the relations between the relevant parameters (eigenvectors) are generally straight lines. (In the magnetospheric context, power-law behavior indicating possible SOC phenomena related to substorms has been conjectured from signatures of the AE spectrum[23a], although the observed characteristics of the spectrum are also amenable to alternative interpretations[23b].) Because there are only a few relevant scaling parameters, we expect that the minimum number of independent invariants for a system at criticality is usually quite small. Since the parameters are related by the physics that characterize the dynamics of the dynamical system, there are definitive relations among the (λ_i/λ_j)'s. These relations are called ''exponent equalities.''

Scaling and similarity. Consider a certain dynamic free energy, F, that scales as the volume of the system. Then, generally, under the scaling transformation of the dynamic renormalization, we have from (13):

$$F(\{V_k\})=e^{d\ell}F(\{e^{\lambda_k\ell}V_k\}), \tag{16}$$

since near criticality, F depends on only the relevant scaling eigenvectors (or parameters). Equation (16) is in the form of a ''generalized homogeneous function.'' It may be put in scaling form as follows:

$$F=V_k^{-d/\lambda_k}Fn(\{I_m\}), \tag{17}$$

where $\{I_m\}$, $m=1,\ldots,n$ represents the minimum number of relevant invariants defined by Eq. (15) and $Fn(\{I_m\})$ is a scaling function that can be determined from experimental observations, simulations or the rigorous solution of the renormalization-group transformation. If $n=1$, then the entire observational data would collapse onto one single curve and the dynamic system is self-similar.

Generally the power laws (15)–(17) for dynamic systems near criticality will deviate from that could be deduced by straightforward dimensional analyses. This is due to the cumulative and combined effects of the long-ranged and multiscaled correlations of the fluctuations near criticality. We may therefore speak of ''anomalous dimensions'' for the stochastic system near self-organized criticality.[3,4] It has become fashionable sometimes to truncate the description of a stochastic system near criticality into a dynamic system expressed in terms of an arbitrarily small number of parameters or ''dimensions'' (i.e., low dimensionality). From the above discussions, it is reasonable to believe that such a prescription is viable (provided one is reasonably sure of what are the relevant parameters to be incorporated in the truncated dynamical equations). When the truncated system exhibits chaoslike behavior, it is then claimed to have a fractal dimension.

All the above results seem to indicate that the magnetotail dynamics (particularly before, during and after substorms) is a stochastic system near forced and/or self-organized criticality.

VII. SYMMETRY BREAKING, CROSSOVER, MEAN FIELD BEHAVIOR, AND THE FLUCTUATION SPECTRA

Sometimes a stochastic system may be perturbed from a particular state of criticality and attracted to another critical state (represented, for example, by another fixed point, L^{**} as shown in Fig. 6). Associated with the second fixed point there generally will be a different set of relevant eigenvalues, eigenvectors, and eigendensities. This type of symmetry breaking is quite common in real physical systems (e.g., tricritical points in metamagnets and He^3–He^4 mixtures[24–27]) and probably occurs for substorm dynamics of the magnetotail, e.g., when the scaling behavior changes from the MHD description to the kinetic description as discussed below in the paragraph on the fluctuation spectra. Under such conditions, the relevant number of scaling directions will change along with the manifestation of new values of anomalous dimensions. As the system evolves from one critical state to another, the renormalization-group transformation can no longer be linearized about a particular fixed point. In the transitional region, the anomalous dimensions change continuously. It is under such conditions that the standard techniques for the determination of fractal dimensions based on methods borrowed from finite-dimensional chaos will generally give questionable results.

Mean field behavior. Sometimes the dynamic system can deviate sufficiently from the critical state. The behavior of the system will generally still be characterized by the invariants (15) and scaling functions of the form given by Eqs. (16) and (17). However, the results can now be deduced by dimensional arguments. The system is then in a so-called ''mean-field'' state. It is possible that the dynamics of the magnetotail may sometimes exhibit mean-field behavior,[3] i.e., the system can still be characterized by a small number of relevant scaling parameters but the dynamical behavior may be understood in terms of simple physical arguments without the detailed analysis of the renormalization-group.

Fluctuation spectra. A standard technique commonly employed to gauge the behavior of intermittent turbulence is through the properties of the spectra of the turbulent fluctuations. For example, in the ''neutral sheet'' region of the magnetotail, one of the more important fluctuation spectra to consider is that of the square of the magnetic fluctuations in the cross-tail direction $\langle \delta B^2 \rangle$. We expect the spectra to generally exhibit either mean-field or fractal characteristics (i.e., nonclassical slopes with discernible deviations from those obtainable by naive dimensional arguments). (See Fig. 8.) In regions where the fluctuations and merging dimensions are much larger than that of the local ion gyroradius, the spectrum is expected to exhibit two distinguishable parts: a domain characterized by the larger scale coherent structures

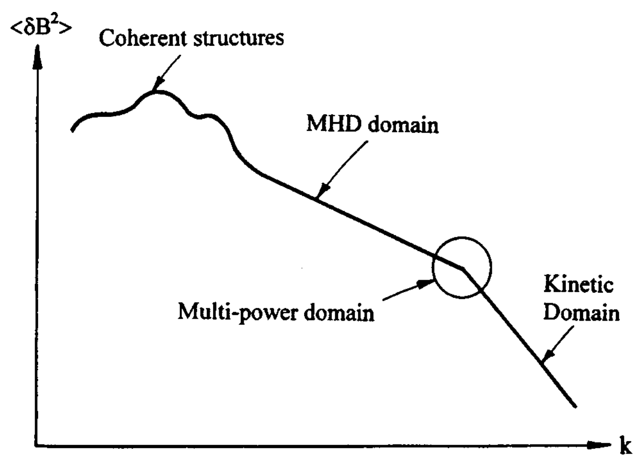

FIG. 8. Fluctuation spectrum near the ''neutral sheet.''

and a domain characterized by the predominantly MHD fluctuations. On the other hand, in regions within the narrow cross-tail current sheet, we expect the spectra to exhibit at least three distinguishable parts: a domain that contains predominantly large scale coherent structures, an MHD domain and a kinetic regime. Such type of fluctuation spectra has been recently observed.[2,28,29] The shapes (slopes) of these spectra have been compared favorably with results based on theoretical scaling ideas.[5–7] The difference of slopes of the various domains of an individual spectrum indicates that the scaling behavior of each domain belongs to a different ''universality class.'' As discussed above, such type of change of scaling behavior from one universality class to another is called ''symmetry breaking.'' In addition to the scaling properties of individual discernible domains, there are also intermediate regimes whose scaling properties are much more complicated (as indicated by the circled region of Fig. 8). The scaling laws for these regions are generally expected to exhibit multiple-power or other nonlinear characteristics. The results generally depend on the details of the symmetry breaking and overlapping behavior among the universality classes and were discussed in detail by Chang and coworkers in a series of papers on critical phenomena.[24–27]

VIII. SUMMARY

In summary, we have introduced a multiscale intermittent turbulence model for the dynamics of the magnetotail. The theory is based on the overlapping resonances of plasma fluctuations. It provides a physical picture of sporadic and localized merging of coherent magnetic structures of varied sizes. Such a picture seems to depict the observational properties of ''bursty bulk flow'' (sporadic localized reconnections) in the magnetotail.[1,2] In this picture, the onset of substorm is due to a global nonclassical nonlinear instability and the dynamics of the magnetotail during the evolution of the substorm is characterized by the phenomenon of forced and/or self-organized criticality. (Such type of critical and noncritical substorm behavior has been simulated based on the cellular automata calculations for a simple analogous one-dimensional sandpile model[30] and the results seem to be quite robust even under strong external driving.[31])

The consequence of this is the prediction of multifractal characteristics of the fluctuation spectra[1,2,5–7,28,29] and the dynamics of the magnetotail behaves essentially as a low-dimensional system. This conclusion seems to agree with the results of some of the recent nonlinear dynamics calculations.[8–13]

In an insightful book, Kennel[32] had suggested that the dynamics of the magnetotail must entail the effects of multiple reconnections and substorm phenomena should be addressed in terms of techniques of modern nonlinear physics. The discussions given in this treatise represent an attempt in this direction.

Postscript. As the reader may have noted, the concept of dynamic criticality for systems far from equilibrium as discussed in this review is more broadly defined than that was originally proposed for self-organized criticality by Bak *et al.*[33] In fact, the concepts of low dimensionality, symmetry breaking, crossover and multifractal spectra described in this paper implicitly require that a small set of parameters is relevant and tuned closed to criticality. Thus, some degree of tuning is generally necessary in the context of our discussions. (See also, discussions of Jensen,[34] Section 6.2.) For these reasons, the author has chosen to use the terminology ''forced and/or self-organized criticality'' instead of simply ''self-organized criticality'' in this review.

ACKNOWLEDGMENTS

I am indebted to V. Angelopoulos, D. Baker, J. Büchner, S. Chapman, H.J. Jensen, J. Kan, C. Kennel, L. Kepko, M. Kivelson, A. Klimas, A.T.Y. Lui, J. Nicoll, H. Petschek, S. Sharma, R. Stenzel, S.W.Y. Tam, D. Tetreault, D. Vassiliadis, D. Vvedensky, N. Watkins, C.C. Wu, and L. Zelenyi for useful discussions. This research was partially supported by AFOSR, NSF, and NASA.

APPENDIX: PATH INTEGRAL OF NONLINEAR STOCHASTIC SYSTEMS

The stochastic behavior of a nonlinear dynamical system such as that is characterized by Eqs. (1)–(7) may be described by the probability density functional $P\{\phi_i\}$, where ϕ_i are the field variables $\{\mathbf{B}, \mathbf{E}, \mathbf{P}, \mathbf{V}, ..., \text{etc.}\}$. It can be shown that $P\{\phi_i\}$ may be expressed as a ''path integral.'' (For analogous quantum mechanical descriptions of path integrals, the readers are urged to consult the classical text, Feynman and Hibbs.[35]) We demonstrate this by following Crew and Chang[36] and consider first the stochastic behavior of a one-degree of freedom system $r(t)$ characterized by the Langevin equation:

$$\dot{r}=u(r(t),t)+\eta(r(t),t), \tag{A1}$$

where $u(r,t)$ is a nonrandom nonlinear force and $\eta(r,t)$ is a random noise.

We consider the path $s(t)$ a particle might take from some initial position $s(0)$ to a final position $s(T)$, and subdivide this into N equal intervals of length $\varepsilon \equiv T/N$. With the notation $s_n \equiv s(n\varepsilon)$, we may rewrite (A1) as

$$s_n=s_{n-1}+\varepsilon u_{n-1}+\varepsilon \eta_{n-1}. \tag{A2}$$

Because of the noise term, there is not a single path $s(t)$, but rather an ensemble of such paths. If we now inquire as to the probability P that the particle is found on some arbitrary path $\{r_n\}$, with $r_0=s_0$, we may compute it from

$$P(\{r_n\})=\langle\delta(s_1-r_1)\cdots\delta(s_N-r_N)\rangle, \tag{A3}$$

where the angular brackets denote an average over the ensemble of path $\{s_n\}$, which by (A2) amounts to a statistical average over the noise at each instant of time. At this point, we use (A2) successively for s_n, taking $n=N$, $N-1,\ldots,1$; in so doing, we note that $s_{n-1}=r_{n-1}$ is required by an adjacent delta function. In this fashion, the nth delta function of (A3) becomes

$$\delta(r_{n-1}+\varepsilon u_{n-1}+\varepsilon\eta_{n-1}-r_n). \tag{A4}$$

We now replace the delta function by the Fourier representation

$$\delta(r)=(2\pi)^{-1}\int dk\,\exp(ikr). \tag{A5}$$

Passing on to the continuum limit, we obtain

$$P\{r\}=\int D[k]\left\langle\exp\left(-i\int_0^T dt\,k(\dot r-u-\eta)\right)\right\rangle, \tag{A6}$$

where we have dropped a constant factor without loss of generality. Thus, we have expressed $P\{r\}$ as a functional integral, the ''path integral.''

We now pass on to the case of the stochastic behavior of a set of field variables, whose dynamics are governed by the generalized Langevin equation:

$$\dot\phi_i=f_i[\underline{\phi}(\underline{x},t),\underline{x},t]+\eta_i(\underline{x},t)\quad(i=1,2,\ldots,n), \tag{A7}$$

where $f_i(i=1,2,\ldots,n)$ are nonrandom forces, $x_\mu(\mu=1,2,\ldots,d)$ and t are the spatial coordinates and time, and $\eta_i(i=1,2,\ldots,n)$ are random stirring forces.

The probability density functional for the stochastic system, $P\{\phi_i\}$, can be calculated in terms of the functional Jacobian, $J(\partial\eta_i/\partial\phi_j)$, prescribed by (A7), using the path integral formalism similar to that discussed above for a single degree of freedom system.[3,4,14] The result is

$$P(\phi_i)=\int D[\pi]\exp\left\{\int dt\,d\underline{x}[iL(\underline{\phi}(\underline{x},t),\underline{\pi}(\underline{x},t),\underline{\dot\phi}(\underline{x},t))]\right\}, \tag{A8}$$

where $\pi_i=\partial L/\partial\dot\phi_i$ may be viewed as the conjugate momenta of the stochastic system, and the stochastic Lagrangian, L, is given by (after integrating over the noise terms)

$$\begin{aligned}L=&\pi_i(\underline{x},t)[\dot\phi_i(\underline{x},t)-f_i]\\&+\frac{i}{2}\int dt'\,d\underline{x}'\,C_{ij}(\underline{x},t;\underline{x}',t')\pi_i(\underline{x},t)\pi_j(\underline{x}',t')\\&+\frac{1}{6}\int dt'\,dt''\,d\underline{x}'\,d\underline{x}''C_{ijk}(\underline{x},t;\underline{x}',t';\underline{x}'',t'')\\&\times\pi_i(\underline{x},t)\pi_j(\underline{x}',t')\pi_k(\underline{x}'',t'')+\cdots,\end{aligned} \tag{A9}$$

where $C_{ij},C_{ijk},\ldots,$ are correlation functions of η_i. From (A9), the cumulants of ϕ's and π's (the correlation and response functions) may be evaluated. Thus the full dynamic behavior of the considered system is fully characterized by the stochastic Lagrangian L. For further discussions, the readers are encouraged to consult Chang *et al.*[14]

[1] V. Angelopoulos *et al.*, J. Geophys. Res. **101**, 4967 (1996).
[2] A. T. Y. Lui, in *Physics of space plasmas, Multiscale Phenomena in Space Plasmas II*, edited by T. Chang and J. R. Jasperse (Center for Theoretical Geo/Cosmo Plasma Physics, Massachusetts Institute of Technology, Cambridge, 1998), vol. 15; p. 233.
[3] T. Chang, IEEE Trans. Plasma Sci. **20**, 691 (1992).
[4] T. Chang, in *Research Trends in Physics: Nonlinear Space Plasma Physics*, edited by H. Alfvén, K. Quest, and R. Z. Sagdeev (American Institute of Physics, New York, 1992), p. 165.
[5] T. Chang, in *Geospace Mass and Energy Flow*, edited by J. L. Horwitz, D. L. Gallagher, and W. K. Peterson (American Geophysical Union Monograph 104, Washington, DC, 1998), p. 193.
[6] T. Chang, in *4th International Conference on Substorms, Japan, 1998*, edited by S. Kokubun and Y. Kamide (Terra Publishing Co., Tokyo and Kluwer Academic Publishers, Dordrecht, 1998), p. 431.
[7] T. Chang, in *Physics of Space Plasmas, Multiscale Phenomena in Space Plasmas II*, edited by T. Chang and J. R. Jasperse (Center for Theoretical Geo/Cosmo Plasma Physics, Massachusetts Institute of Technology, Cambridge, 1998), vol. 15, p. 61.
[8] D. N. Baker, A. J. Klimas, R. L. McPherron, and J. Büchner, Geophys. Res. Lett. **17**, 41 (1990).
[9] A. J. Klimas, D. N. Baker, and D. A. Roberts, Geophys. Res. Lett. **18**, 1635 (1991).
[10] A. J. Klimas, D. N. Baker, D. A. Roberts, D. H. Fairfield, and J. Büchner, J. Geophys. Res. **97**, 12253 (1992).
[11] D. A. Roberts, D. N. Baker, A. J. Klimas, and L. F. Bargatze, Geophys. Res. Lett. **18**, 151 (1991).
[12] D. V. Vassiliadis, A. S. Sharma, T. E. Eastman, and K. Papadopoulos, Geophys. Res. Lett. **17**, 1841 (1990).
[13] A. S. Sharma, D. V. Vassiliadis, and K. Papadopoulos, Geophys. Res. Lett. **20**, 335 (1993).
[14] T. Chang, D. D. Vvedensky, and J. F. Nicoll, Phys. Rep. **217**, 279 (1992); T. Chang, J. F. Nicoll, and J. E. Young, Phys. Lett. **67A**, 287 (1978).
[15] J. Taylor, Phys. Rev. Lett. **33**, 1139 (1974).
[16] D. Tetreault, J. Geophys. Res. **97**, 8531 (1992).
[17] A. T. Y. Lui, J. Geophys. Res. **101**, 4899 (1996).
[18] J. Büchner, in *4th International Conference on Substorms, Japan, 1998*, edited by S. Kokubun and Y. Kamide (Terra Publishing Co., Tokyo and Kluwer Academic Publishers, Dordrecht, 1998), p. 461.
[19] A. A. Galeev, M. M. Kuznetsova, and L. M. Zelenyi, Space Sci. Rev. **44**, 1 (1986).
[20] H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, Cambridge, England, 1971).
[21] K. G. Wilson, Sci. Am. **241(2)**, 158 (1979).
[22] K. G. Wilson and J. B. Kogut, Phys. Rep. **12C**, 75 (1974).
[23] (a) G. Consolini, in *Proceedings of the ''Cosmic Physics in the Year 2000,'' Bologna, 1997*, edited by S. Aiello, N. Iucci, G. Sironi, A. Treves, and U. Villante (Societa Italiana de Fisica, Italy, 1997), Vol. 58, p. 123; (b) V. M. Uritsky and M. I. Pudovkin, Ann. Geophys. (Germany) **16**, 1580 (1998).
[24] T. Chang and H. E. Stanley, Phys. Rev. B **8**, 4435 (1973).
[25] T. Chang, A. Hankey, and H. E. Stanley, Phys. Rev. B **8**, 346 (1973); *ibid.* B **7**, 4263 (1973).
[26] J. F. Nicoll, T. Chang, and H. E. Stanley, Phys. Rev. Lett. **34**, 1446 (1974).
[27] J. F. Nicoll, T. Chang, and H. E. Stanley, Phys. Rev. Lett. **36**, 113 (1976).
[28] M. Hoshino, A. Nishida, T. Yamamoto, and S. Kokubun, Geophys. Res. Lett. **21**, 2935 (1994).
[29] A. V. Milovanov, L. M. Zelenyi, and G. Zimbardo, J. Geophys. Res. **101**, 19903 (1996).
[30] S. C. Chapman, N. W. Watkins, R. O. Dendy, P. Helander, and G. Rowlands, Geophys. Res. Lett. **25**, 2397 (1998).

[31]N. W. Watkins, S. C. Chapman, R. O. Dendy, and G. Rowlands, Geophys. Res. Lett. **26**, 2617 (1999).
[32]C. Kennel, *Convection and Substorms: Paradigms of Magnetospheric Phenomenology* (Oxford University Press, Cambridge, England, 1995).
[33]P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
[34]H. J. Jensen, *Self-Organized Criticality* (Cambridge University Press, Cambridge, England, 1998).
[35]R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw Hill, New York, 1965).
[36]G. Crew and T. Chang, Phys. Fluids **31**, 3425 (1988).