GEOPHYSICAL RESEARCH LETTERS, VOL. 32, L05109, doi:10.1029/2004GL021445, 2005

Intermittency analyses on the SIERRA measurements of the electric field fluctuations in the auroral zone

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Received 8 September 2004; revised 20 January 2005; accepted 28 January 2005; published 12 March 2005.

[1] We perform intermittency analyses on the electric field data obtained by the SIERRA sounding rocket in the auroral zone. The electric field fluctuations are broadband, covering the extremely low-frequency range with a power-law relation, similar to the type of fluctuations commonly observed at various altitudes of the auroral region. Our preliminary analyses of the data based on the technique of probability distribution functions indicate that the electric field fluctuations are intermittent in the spacecraft frame. Using the methods of wavelet analyses and local intermittency measures, we determine the degree of intermittency of the fluctuations at various scales. It is found that the electric field fluctuations are more intermittent at smaller scales. **Citation:** Tam, S. W. Y., T. Chang, P. M. Kintner, and E. Klatt (2005), Intermittency analyses on the SIERRA measurements of the electric field fluctuations in the auroral zone, *Geophys. Res. Lett.*, *32*, L05109, doi:10.1029/2004GL021445.

1. Introduction

[2] Broadband extremely low-frequency (BB-ELF) electric field fluctuations have often been observed in the auroral zone [*Kintner et al.*, 2000]. It is commonly believed that such fluctuations are responsible for the acceleration of the auroral ions [*André et al.*, 1998]. *Chang et al.* [1986] suggested that the left-hand polarized component of the electric field fluctuations may transversely energize the ions via cyclotron resonance. They also suggested that when the ions travel upward along the field lines, their perpendicular energy will be transferred to the parallel direction due to the mirror effect, thus leading to a conic distribution which had been observed by the DE-1 satellite [*Winningham and Burch*, 1984]. Based on this theory, *Retterer et al.* [1987] demonstrated the formation of oxygen-ion conic distributions that resembled the DE-1 observations, thus explaining the association between the BB-ELF electric field fluctuations and the transverse energization of the auroral ions. BB-ELF electric fields accompanied by events of transversely accelerated auroral ions as measured by the Freja satellite were interpreted by *Norqvist et al.* [1996] to be electromagnetic. Recently, there have been increasing experimental indications that these fluctuations in the auroral zone are predominantly electrostatic [*Kintner et al.*, 1996].

[3] Our understanding of BB-ELF fluctuations in the auroral zone has been enhanced mainly by the recent developments in data analyzing techniques. For example, by determining the interferometric coherency of the auroral fluctuating electric fields, *Kintner et al.* [2000] were able to estimate the range of the wavelengths of these fluctuations if the fluctuations were waves. They found that for fluctuations near the oxygen gyrofrequency, the coherency was small. With both the ranges of frequencies and wavelengths known, they concluded that such fluctuations do not correspond to any normal wave modes of a homogeneous plasma.

[4] It has been suggested by *Chang* [2001] that the broadband power spectrum signature of the BB-ELF fluctuations might be the manifestation of intermittent turbulence. The origin of intermittent fluctuations in magnetized plasmas was interpreted [*Chang et al.*, 2004, and references therein] as the result of the mixing and/or interactions of sporadic and localized coherent structures. Such coherent structures may take on the forms of spatial irregularities, density depletions, convective structures, electron and ion holes, double layers and various types of solitary waves and have been described as resonant or non-propagating in differentiation from those that are non-resonant and satisfy the plasma dispersion relations. When detected in the spacecraft frame the signatures of such structures are Doppler shifted [*Temerin*, 1979; *Stasiewicz et al.*, 2000; *Bonnell et al.*, 2001; *Angelopoulos et al.*, 2001] and therefore may be more clearly resolved in terms of multi-spacecraft observations. In the auroral zone, a significant fraction of such fluctuations is expected to be electrostatic and transverse [*Chang*, 2001], perhaps intermixed with a component of electrostatic [*Bonnell et al.*, 1996] or electromagnetic waves.

[5] *Chang et al.* [2004] have investigated the effect of intermittent fluctuations on the acceleration of auroral O^+ ions. They have shown that the degree of intermittency is an important factor determining the efficiency of the ion energization.

[6] The goal of this study is to analyze the intermittency of the BB-ELF electric field fluctuations in the auroral zone, as measured in the Sounding of the Ion Energization Region: Resolving Ambiguities (SIERRA) mission. We shall present preliminary results of the data analyses based on the techniques of probability distribution functions (PDF), wavelet analyses and local intermittency measures (LIM).

2. Electric Field Instrument and Data

[7] SIERRA consisted of a main payload and two identical sub-payloads separated by hundreds of meters in a triangle configuration within a plane perpendicular to the

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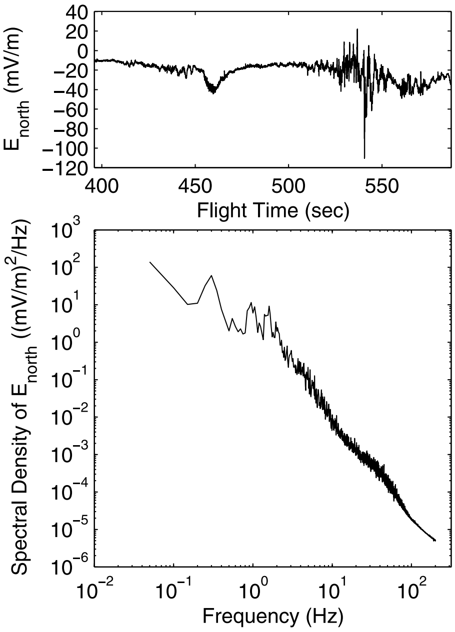


Figure 1. Top: Plot of electric field component E_{north} versus the flight time of SIERRA for the duration when the rocket was above 700 km altitude. Bottom: Average spectral density of E_{north} over the duration.

geomagnetic field. The payloads reached an apogee of 735 km altitude. Their speed was approximately 1.3 km/s perpendicular to the magnetic field. The electric field data used in this study were measured by the FWD sub-payload, which spun about an axis perpendicular to the geomagnetic field with a period of 0.6 second. The sub-payload used a pair of orthogonal 6-meter COWBOYS (COrnell Wire-BOom Yo-yo System) antennas spanning a plane perpendicular to the spin axis to measure the electric fields with a 1-millisecond resolution. For these analyses at frequencies both above and below the O^+ cyclotron frequency, we assume that the electric fields were primarily transverse to the geomagnetic field implying that the parallel electric field was small even though other measurements at higher frequencies, mostly above the H^+ cyclotron frequency, implied some parallel component to the BB-ELF fluctuations [*Bonnell et al.*, 1996]. After the elimination of the spin effect of the sub-payload and an application of a low-pass filter at 200 Hz, the data reduce to a time series of an electric field component, which is perpendicular to the geomagnetic field. The component is approximately in the geographical north direction and labeled E_{north}.

[8] Figure 1 (top) shows the E_{north} fluctuations during the time when SIERRA was above 700 km altitude. By averaging over 1910 spectra, we have found the spectral density for E_{north} during this time interval, as shown in Figure 1 (bottom). Notice that the fluctuations are broadband in nature, with power-law spectral density covering the extremely low-frequency range (3 to 200 Hz). Such a spectrum suggests that the fluctuations are typical BB-ELF electric fields, which have been frequently observed at different altitudes of the auroral zone. We have also taken the average spectral density of the fluctuations over different sub-intervals, and found similar power laws in the extremely low-frequency range for all those time ranges. Thus, the results are quite robust.

3. Probability Distribution Functions (PDF)

[9] To examine the electric field fluctuations for intermittency, we first study how much their statistics deviate from those of Gaussianity. The amount of the deviation indicates the degree of intermittency. For a fluctuating field X, we may generate the PDF $P(\delta X, \tau)$ of $\delta X \equiv X(t + \tau) - X(t)$ for different values of τ over a certain range of time t. It is expected that the range of δX is wider for larger values of τ. Thus, in order to compare the deviation of the PDF from Gaussianity at different scales, it is useful to consider their normalized distribution instead. We normalize the PDF by $\sqrt{\langle(\delta X)^2\rangle}$, the root-mean-square value of δX, which we denote as $\sigma(\tau)$. The normalized PDF becomes:

$$P_*(\delta X/\sigma, \tau) = \sigma P(\delta X, \tau), \qquad (1)$$

whose variance is always unity, independent of the scale τ. Based on the fluctuations shown in Figure 1 (top), we have generated normalized PDF P_* with a few different values of τ. Figure 2 shows the distributions P_* at three different scales for $X = E_{north}$. We find that P_* does not differ by much for τ up to about 1 second. For comparison, we have also generated Gaussian fluctuations, and found the corresponding P_*, which is independent of τ, and shown in Figure 2. The significant deviation of the experimental data from Gaussianity indicates intermittency. Recently, *Hnat et al.* [2002] has introduced a mono-power scaling relation of the form $P(\delta X, \tau) = \tau^{-\gamma} P_s(\delta X \tau^{-\gamma}, \tau)$. We note that the mono-power scaling condition implies $\sigma \sim \tau^{\gamma}$ and the collapse of P_* onto a single curve, and *vice versa*. We find that $\sigma \sim \tau^{\gamma}$ holds true in our data for scales up to about 200 ms with $\gamma = 1.60 \pm 0.03$. Thus, the mono-power scaling relation should also hold true for the data within the same scale range.

[10] To characterize the intermittency, we try to fit the normalized PDF with a Castaing distribution [*Castaing et al.*, 1990]:

$$\Pi_\lambda(\xi) = \frac{1}{2\pi\lambda}\int_0^\infty \frac{d\alpha}{\alpha^2}\exp\left(\frac{-\xi^2}{2\alpha^2}\right)\exp\left(\frac{-\ln^2(\alpha/\alpha_0)}{2\lambda^2}\right), \qquad (2)$$

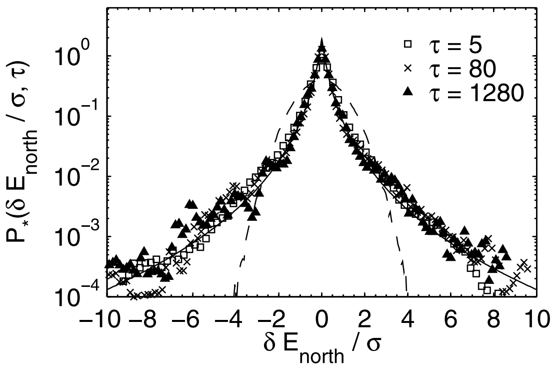
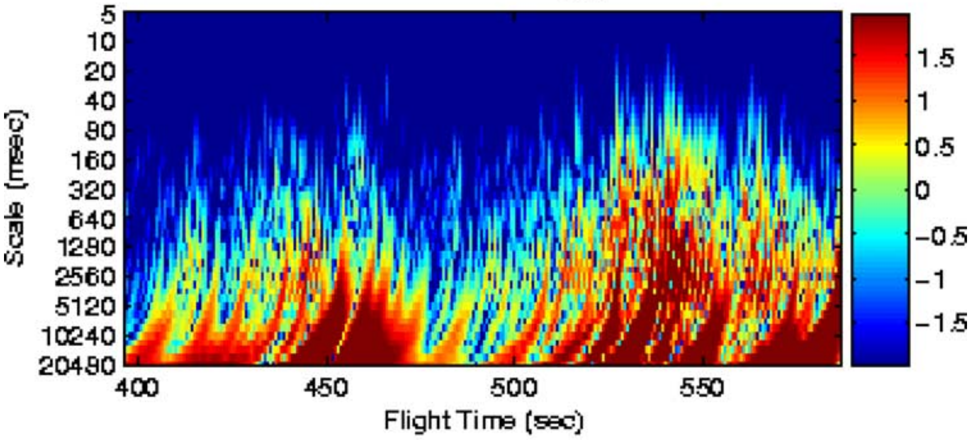


Figure 2. Normalized PDF $P_*(\delta E_{\mathrm{north}}/\sigma, \tau)$ at τ = 5, 80 and 1280 ms. The solid line corresponds to the normalized Castaing distribution with λ = 1.02. The results of the analyses for Gaussian fluctuations (dashed) are shown for comparison.


a) Logarithm of Power of E_{north} Wavelets

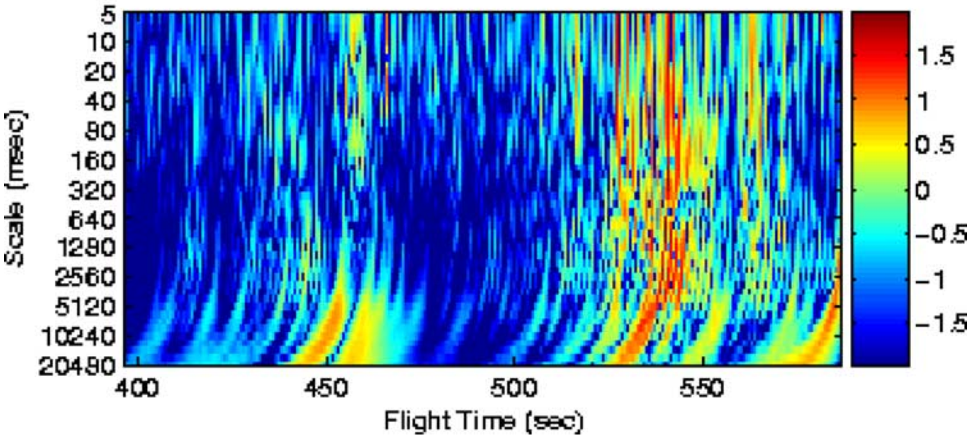

b) $\log_{10}$ LIM for E_{north} Wavelets

Figure 3. (a) Power of the wavelets for E_{north}; (b) *LIM* for the wavelets.

where $\lambda > 0$ is the parameter that characterizes the intermittency of the distribution. For the distribution in equation (2) to have a variance of unity, it is necessary that $\ln \alpha_0 = -\lambda^2$. For $\lambda = 0$, equation (2) reduces to a Gaussian distribution. As λ increases, the degree of intermittency increases, and $\Pi_\lambda(\xi)$ becomes larger at small and large values of ξ. P_* can be approximated by a Castaing distribution with $\lambda = 1.02 \pm 0.05$ based on a minimization of χ^2 (Figure 2).

4. Wavelet Analyses and Local Intermittency Measures

[11] As the electric field fluctuations are intermittent, we identify the time at which the power of the fluctuations at various scales concentrates. With such information, we can determine the intermittency of the fluctuations at these scales. This can be accomplished by the technique of Local Intermittency Measures (LIM) using wavelet transforms. A wavelet transform is generally composed of modes which are square integrable localized functions that are capable of unfolding fluctuating fields into time (or space) and scales [*Farge*, 1992]. The power of the fluctuations at different time and scales is characterized by a set of coefficients of the transform. We apply the Haar wavelet transform [*Bruno et al.*, 2001] to find $W_s(t)$, the coefficients of the wavelets for E_{north} at various scales s and time t. The power of the wavelets $|W_s(t)|^2$ is shown in Figure 3a. The strong fluctuations around $t = 540$ (see Figure 1) contribute to the high power of the wavelets in this time range. The power of the fluctuations mainly concentrates at the scales of 320 ms or larger.

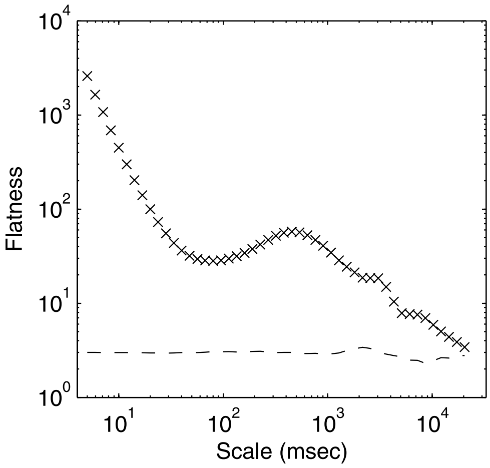


Figure 4. Plot of the flatness for the wavelets versus scale: symbols “×” for E_{north} and dashed line for Gaussian fluctuations.

[12] To study the intermittency of the fluctuations at different scales, we first define $LIM(s,\ t) \equiv |W_s(t)|^2/\langle|W_s(t)|^2\rangle_t$, where $\langle\ldots\rangle_t$ denotes the averaging over time. *LIM* measures the relative power among wavelets of the same scales throughout the time domain. For any given scale, *LIM* averages to 1 over time. From the results shown in Figure 3b, although there is strong wavelet power near $t = 540$ at the scales larger than 320 ms (see Figure 3a), the *LIM* in this time range is large only for scales between 320 and 5120 ms, but considerably smaller for the larger scales. This is because at other time, there is also strong wavelet power at these larger scales. In other words, the power of the wavelets at these scales is more uniform in time compared with the smaller scales. In fact, for a given scale, intermittency is not measured by its overall wavelet power, but is characterized by the degree of non-uniformity of the wavelet power (or *LIM*) in time; the less uniform the *LIM*, the higher degree the intermittency. A direct measure of the degree of intermittency is flatness, $F(s) \equiv \langle[LIM(s,\ t)]^2\rangle_t$ [*Meneveau*, 1991]. We note that for Gaussian fluctuations $F = 3$. Therefore, $F > 3$ would indicate that the fluctuations are intermittent. The flatness for E_{north} as well as that for Gaussian fluctuations at various scales is shown in Figure 4. Except at the largest scales, the flatness for E_{north} is generally larger than 3. The flatness is generally higher for smaller scales, indicating that the degree of intermittency for the electric field fluctuations varies inversely with the time scale. For time scales of a few milliseconds, the flatness increases to several thousands.

5. Conclusion

[13] We have performed intermittency analyses for the electric field fluctuations in the auroral zone, as measured by the SIERRA rocket. The spectrum for the fluctuations is broadband, and can be described by a power-law relation over the extremely low-frequency range, suggesting that the fluctuations are those of the BB-ELF electric fields, which have frequently been observed in the auroral zone.

[14] By examining the PDF of the electric field difference at various scales of separation, we have concluded that large fluctuations are more probable than one would expect from normal distributions, a typical characteristic of intermittent turbulence. We demonstrate the sporadic and localized nature of the fluctuations using the techniques of wavelet transform and local intermittency measure. The results based on the flatness indicate that the degree of intermittency of the electric field generally decreases with the time scale. The fluctuations are highly intermittent at scales of a few milliseconds. These results strongly support the theory of *Chang* [2001] and *Chang et al.* [2004], indicating that the underlying physical process of BB-ELF is the intermixing and nonlinear interactions of sporadic and localized coherent structures that produce large intermittent plasma fluctuations, combined with a small fraction of electrostatic and electromagnetic waves. If the coherent structures are composed primarily of stationary or slowly convective structures, then the fluctuations would appear as Doppler-shifted irregularities. However, this does not exclude the possibility of the entrainment of a small percentage of other types of coherent structures and propagating modes. Such type of intermittent turbulence is a phenomenon of forced and/or self-organized criticality and complexity, which naturally lead to power-law spectral densities.

[15] **Acknowledgment.** This work is partially supported by NASA and NSF.

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