Memo to: MIT XIS Group
From: John Doty
Date: June 9, 2002
Subject: TEC Selection

■ The Model

I'm using the model I used for the initial TEC selection for ASTRO−E1.This model is much more accurate than commonly used graphical techniques. It correctly reproduced the known behavior of the ASCA cooler, and predicted the ASTRO−E1 prototype performance with reasonable accuracy. It incorporates a reasonable set of variables as variables: heat flows vary with temperature, and temperatures vary with heat flows. Its results are therefore self−consistent.It is purely algebraic, so there are no convergence issues, although the algebra is rather complicated (focal plane temperature is related to TEC current by a sixth order polynomial with complicated coefficients). But that's what computers are for.

The basic static model:

q is the heat flow into the cold end of the TEC.
i is the current.
p0 is the CCD electrical dissipation.
ch is the thermal conductance from housing to focal plane.
th is the housing temperature.
tc is the focal plane temperature.
t0 is the heat sink temperature.
thetah is the thermal resistance from housing to heat sink.

```
model = {
    q == g i − c deltat − alpha r i^2,
    q == p0 + ch deltat,
    th == t0 + thetah ( i^2 r + p0 ),
    deltat == th − tc };
```

Relation of the Marlow catalog parameters to the physical TEC parameters in the above model:

```
MarlowRules = {
   g → 2 qmax (th / tspec) ^ 2 / imax,
   c → qmax / deltatmax,
   alpha → qmax (th / tspec) ^ 2 / (imax vmax),
   r → vmax / imax
  };
```

Solve for cold side temp, using the model above:

```
tc[i_] = tc /. Solve[model //. MarlowRules, tc, {q, th, deltat}][[1]]
```

$$\frac{1}{\text{ch} + \frac{\text{qmax}}{\text{deltatmax}}}$$
$$\left(\text{p0} + \text{ch t0} + \frac{\text{qmax t0}}{\text{deltatmax}} + \text{ch p0 thetah} + \frac{\text{p0 qmax thetah}}{\text{deltatmax}} + \frac{\text{i}^2\ \text{qmax t0}^2}{\text{imax}^2\ \text{tspec}^2} - \frac{2\ \text{i qmax t0}^2}{\text{imax tspec}^2} + \right.$$
$$\frac{2\ \text{i}^2\ \text{p0 qmax t0 thetah}}{\text{imax}^2\ \text{tspec}^2} - \frac{4\ \text{i p0 qmax t0 thetah}}{\text{imax tspec}^2} + \frac{\text{i}^2\ \text{p0}^2\ \text{qmax thetah}^2}{\text{imax}^2\ \text{tspec}^2} - \frac{2\ \text{i p0}^2\ \text{qmax thetah}^2}{\text{imax tspec}^2} +$$
$$\frac{\text{ch i}^2\ \text{thetah vmax}}{\text{imax}} + \frac{\text{i}^2\ \text{qmax thetah vmax}}{\text{deltatmax imax}} + \frac{2\ \text{i}^4\ \text{qmax t0 thetah vmax}}{\text{imax}^3\ \text{tspec}^2} -$$
$$\frac{4\ \text{i}^3\ \text{qmax t0 thetah vmax}}{\text{imax}^2\ \text{tspec}^2} + \frac{2\ \text{i}^4\ \text{p0 qmax thetah}^2\ \text{vmax}}{\text{imax}^3\ \text{tspec}^2} -$$
$$\left.\frac{4\ \text{i}^3\ \text{p0 qmax thetah}^2\ \text{vmax}}{\text{imax}^2\ \text{tspec}^2} + \frac{\text{i}^6\ \text{qmax thetah}^2\ \text{vmax}^2}{\text{imax}^4\ \text{tspec}^2} - \frac{2\ \text{i}^5\ \text{qmax thetah}^2\ \text{vmax}^2}{\text{imax}^3\ \text{tspec}^2}\right)$$

Cold side temp as a function of power:

```
tcp[ p_ ] = Simplify[tc[ imax Sqrt[ p /( imax vmax) ]]]
```

$$\left(\text{imax qmax (t0 + (p + p0) thetah) tspec}^2\ \text{vmax} + \right.$$
$$\frac{1}{\sqrt{\frac{\text{p}}{\text{imax vmax}}}}\left(\text{deltatmax}\left(\text{p}^3\ \text{qmax thetah}^2\left(-2 + \sqrt{\frac{\text{p}}{\text{imax vmax}}}\right) + \right.\right.$$
$$2\ \text{p}^2\ \text{qmax thetah (t0 + p0 thetah)}\left(-2 + \sqrt{\frac{\text{p}}{\text{imax vmax}}}\right) + \text{imax (p0 + ch t0 + ch p0 thetah)}$$
$$\text{tspec}^2\sqrt{\frac{\text{p}}{\text{imax vmax}}}\ \text{vmax} + \text{p}\left(\text{qmax (t0 + p0 thetah)}^2\left(-2 + \sqrt{\frac{\text{p}}{\text{imax vmax}}}\right) + \text{ch imax}\right.$$
$$\left.\left.\left.\left.\text{thetah tspec}^2\sqrt{\frac{\text{p}}{\text{imax vmax}}}\ \text{vmax}\right)\right)\right)\Big/\ \text{(imax (ch deltatmax + qmax) tspec}^2\ \text{vmax)}$$

The model uses absolute temperature. For plotting, Celsius is more convenient:

```
tcpc[p_] := tcp[p] - 273
```

Also get the hot side temperature:

```
th[i_] = th /. Solve[model //. MarlowRules, th, {q, tc, deltat}][[1]]
thp[p_] = th[imax Sqrt[p / (imax vmax)]]
thpc[p_] := thp[p] - 273
```

$$\text{t0} + \text{p0 thetah} + \frac{\text{i}^2\ \text{thetah vmax}}{\text{imax}}$$

$$\text{t0} + \text{p thetah} + \text{p0 thetah}$$

■ ASTRO-E2

Model parameters guessed for ASTRO−E.For consistency all temperatures are kelvins, heat flows and electrical dissipations are watts:

```
e2nom = {
   p0 → 0.02,
   ch → 0.003,
   thetah → 1.0,
   t0 → 273 - 45
  };
```

The original ASTRO−ETEC design (kelvins, watts, amps, volts):

```
MI3026x3 = {
   deltatmax → 109,
   qmax → 3 * 0.62,
   imax → 1.3,
   vmax → 3 * 3.4,
   tspec → 300
  };
```

Al's favorite TEC with all the elements:

```
AP100 = {
   deltatmax → 67.,
   qmax → 3.2,
   imax → 1.9,
   vmax → 12.5,
   tspec → 273 - 40
  };
```

Al's favorite TEC with 70% of the elements:

```
AP70 = {
   deltatmax → 67.,
   qmax → 0.7 * 3.2,
   imax → 1.9,
   vmax → 0.7 * 12.5,
   tspec → 273 - 40
  };
```

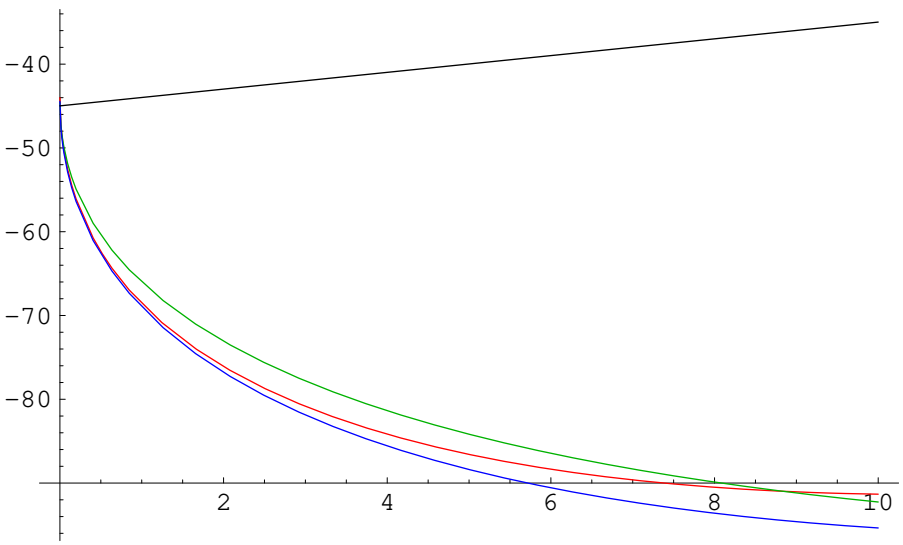
■ Results

DEfine a handy plotting function:

```
Needs["Graphics`Colors`"]
visibleGreen = RGBColor[0, .7, 0];
plot[ fpmod_] :=
 Plot[Evaluate[{tcpc[p] /. MI3026x3, tcpc[p] /. AP100, tcpc[p] /. AP70, thpc[p]} /. fpmod],
  {p, 0, 10} , {PlotStyle → {{Red}, {visibleGreen}, {Blue}, {Black}}}]
```

Plot the results: cold side temperatures are in color. The original multi−TECdesign is red, the custom TEC is green, and 70% of the custom TEC is blue. For all cases, the hot side temperature is black. The vertical axis is temperature (C), the horizontal is electrical power input (W).

```
plot[e2nom]
```




```
- Graphics -
```

Numeric values for 5 and 10 w of electrical power in:

```
tcpc[5] /. e2nom /. MI3026x3
```

```
-86.5815
```

```
tcpc[5] /. e2nom /. AP100
```

```
-84.1717
```

```
tcpc[5] /. e2nom /. AP70
```

```
-88.3839
```

```
tcpc[10] /. e2nom /. MI3026x3
```

```
-91.3274
```

```
tcpc[10] /. e2nom /. AP100
```

```
-92.2618
```

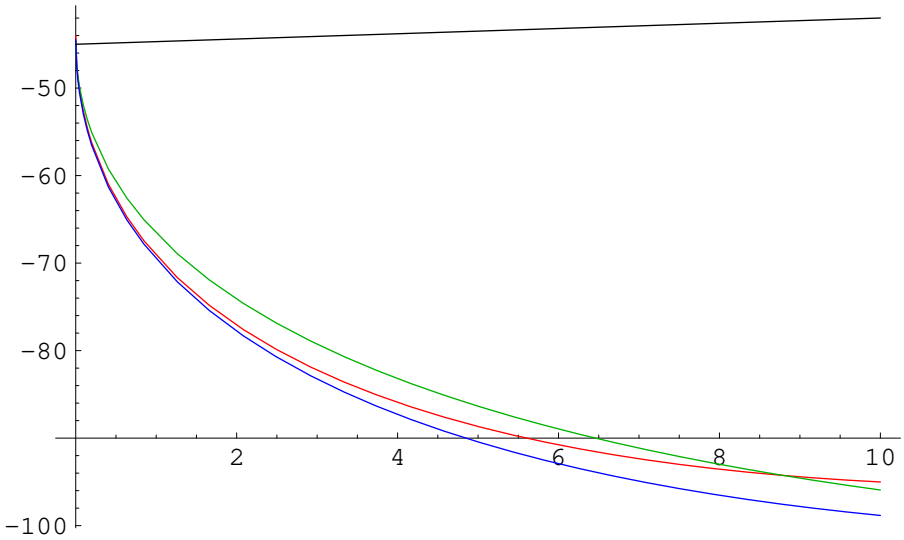
```
tcpc[10] /. e2nom /. AP70
```

```
-95.3601
```

Colder parameters (assumes less thermal resistance to the radiator):

```
e2cold = {
   p0 → 0.02,
   ch → 0.003,
   thetah → 0.3,
   t0 → 273 - 45
  };
```

```
plot[e2cold]
```

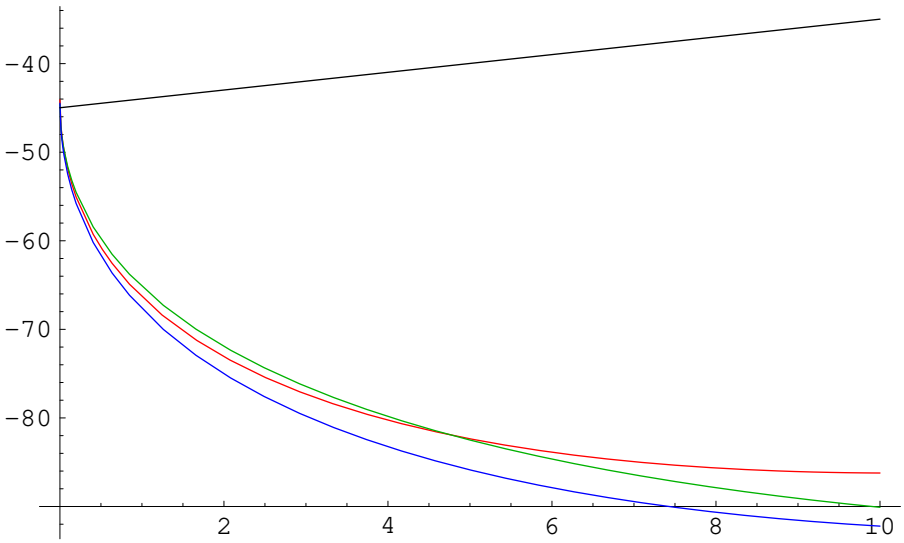



```
- Graphics -
```

Warmer parameters (more heat load to focal plane and more resistance to radiator):

```
e2hot = {
    p0 → 0.02,
    ch → 0.005,
    thetah → 1.0,
    t0 → 273 - 45
   };
```

```
plot[e2hot]
```




```
- Graphics -
```

■ Other Parameters

If you have this memo in *Mathematica* notebook form, you can plug in your own favorite parameters and compute the results with *Mathematica*. Alternatively, just send me the parameters.

■ Analysis

The first thing to notice is that the curves are close together. I've seen this before: in these designs any TEC that might pausibly be considered near optimum yields near optimum performance.

I prefer to focus on the temperature achieved at a given power rather than the power required to achieve a given temperature. The reason is that the power required is a touchy function of poorly known parameters, while the temperature is not. Also, the available power (5W early, 10W later) is defined by the capacity of the spacecraft power system, while the −90 degree temperature goal is a guess at what might be nice to have. Note that the ASCA SIS worked (although not as well as we wanted) at −60:from that perspective the differences in these designs are small.

That said, the smaller custom design ("AP70") is clearly the winner here: it's 2−4degrees better than the bigger custom device ("AP100").

The big issue is ruggedness: I'd rather have four focal planes at −87than three at −91.However, the TEC in this design is not a load bearing element, so there isn't any simple reason to expect a higher stress level in the smaller TEC than in the larger one. It depends on the details, but given that the focal plane is supported independently I'd expect the strain to be independent of the TEC size.